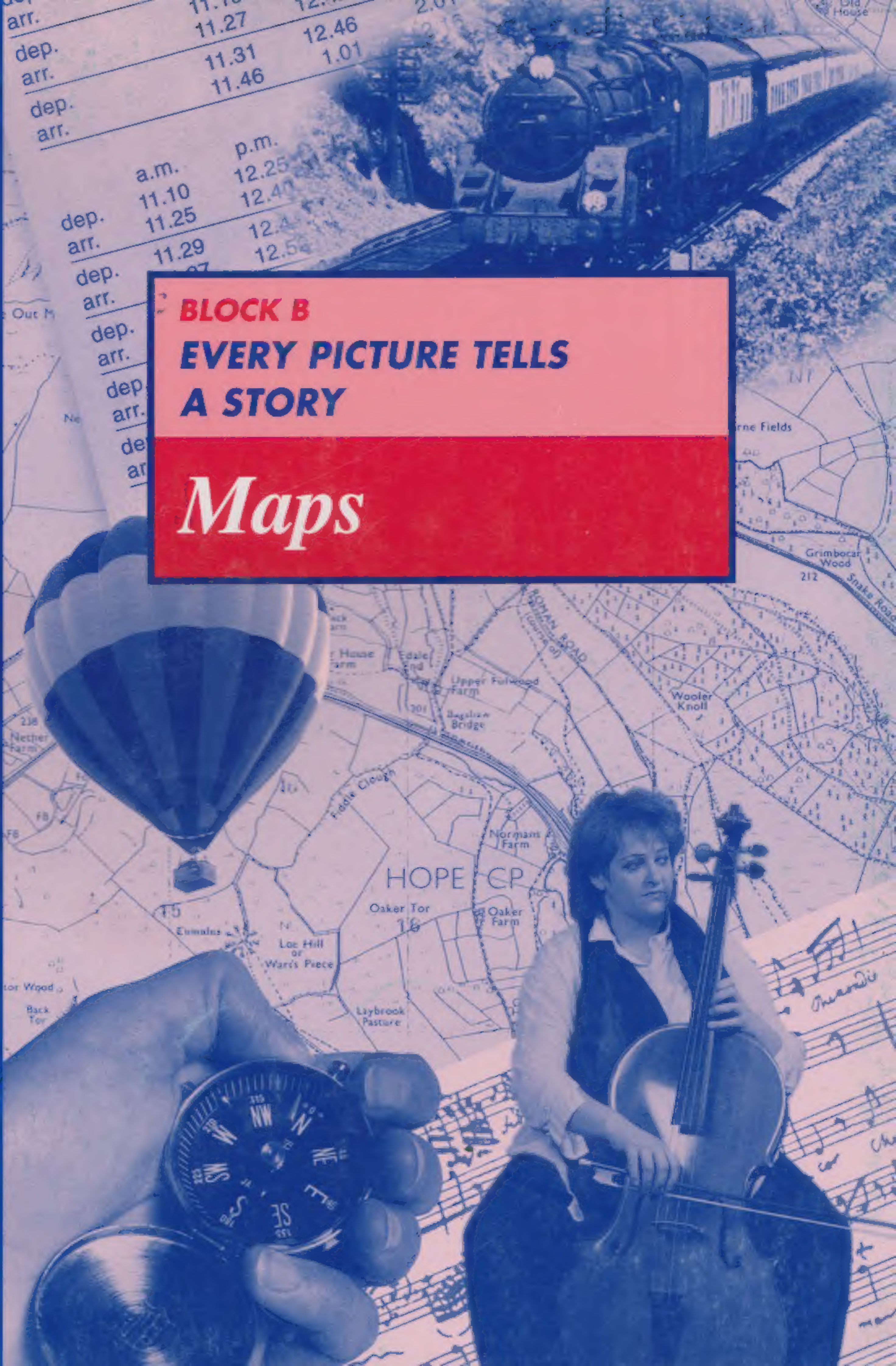
MU120 Unit 6

The Open University

Mathematics and Computing
A first level multidisciplinary course

Open Mathematics



BLOCK B

EVERY PICTURE TELLS A STORY

Maps

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Maps

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The Open University, Walton Hall, Milton Keynes, MK7 6AA.

First published 1996. Reprinted 1998, 1999, 2000.

Edited, designed and typeset by The Open University, using the Open University TEX System.

Printed and bound in the United Kingdom by Henry Ling Ltd, at the Dorset Press, Dorchester, Dorset.

ISBN 0 7492 2232 8

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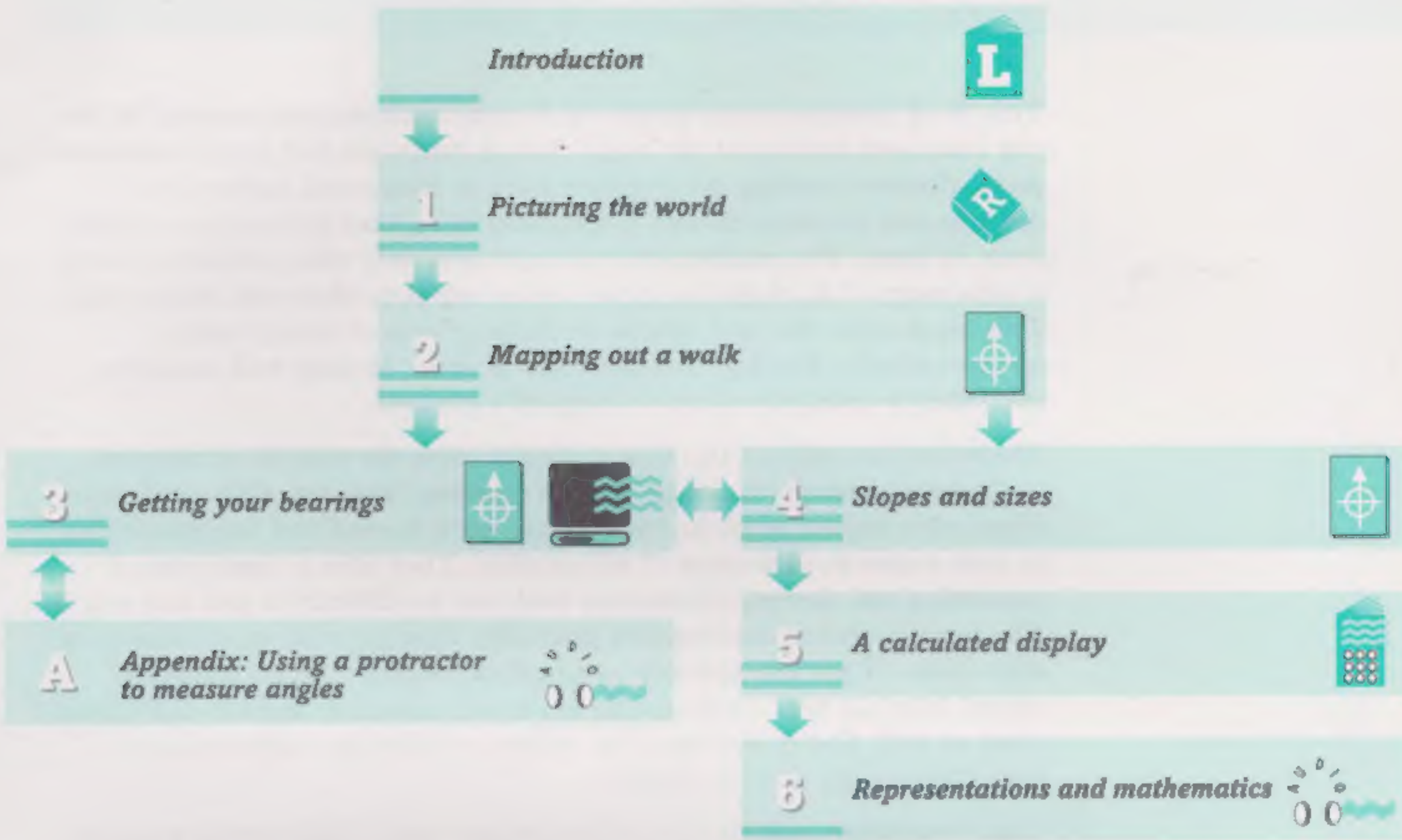
Contents

Study guide

As part of your learning for *Unit 6*, there is an extra component—an extract from an Ordnance Survey (OS) map. You will mainly be using the map during your study of Sections 2 and 3, but you may find it useful during Section 1. The unit contains six sections and Section 1 asks you to consider your own knowledge and use of maps. Three short reader articles are integrated into this first section. You could read the articles at any time during an early study session.

Sections 2 and 3 may take a longer time to study than the other sections, especially if you are unaccustomed to map reading, and much of your work will involve activities using the OS map extract. You will need to lay the map on a flat surface and be in an area where there is good lighting. Studying Section 3 includes a video band, 'Getting your bearings', and also involves some use of the map. Ideally, you should watch this before you begin to study the section. You will need a protractor to complete some of the exercises and there is an audiotape sequence associated with this—for those not familiar with using this measuring device.

Section 4 introduces you to the technique of drawing profiles and graphs and you will begin to use graphs to explain and interpret relationships. You will need centimetre graph paper for this section and the OS map. Using your calculator to draw profiles and graphs is the focus of Section 5. The unit ends with Section 6, which includes a second audio sequence and a Learning File activity, which consolidates your responses to earlier activities for this activity.



Summary of sections and other course components needed for *Unit 6*

Introduction to Block B

This block presents a new viewpoint in your mathematical studies. So far, you have been looking at the world from a statistical and largely numerical point of view—working out numbers such as means and medians to describe and characterize sets of data and using that information to make sense of them. But mathematics does not deal only with numbers: it uses a wide range of symbols to express and manipulate ideas and relationships. This block takes the next step in developing ways of seeing things mathematically. Block A was particularly about looking with numbers; this block is primarily about looking with symbols.

There are four units in this block. *Unit 6* takes the context of maps to explore the symbolic language used to represent features of the landscape. Maps are complex symbols, but you can learn to read and interpret them to gain access to their store of information. They offer a visual way of presenting and sharing information that may be difficult to put into words. The same is true in mathematics generally. Sharing mathematical ideas is often easier if you use symbolic or graphical representations instead of words. It is not that words are no use at all, indeed we need to talk about ideas as well. But in mathematics, words are often less appropriate to conveying clearly what is desired.

Unit 7 considers another form of symbolism—that displayed by graphical presentation of information. In mathematics, graphs are used to give a visual impression of the relationship between two things, and are drawn according to certain conventions. To interpret a graph—and to draw or display useful graphs of your own on your calculator—you need to understand and practise graphing and working with conventions.

Recall in *Unit 1* that generalizing relationships is one of the central features of mathematics. It is one of the things that mathematicians do.

Unit 8 introduces algebra. Algebra uses letters and other symbols to describe general mathematical relationships. Looking at relationships algebraically is a way of moving from a specific to a more general case: a way of looking at how calculations will go in general rather than working out a specific numerical result. Using symbols algebraically provides a way of writing down relationships that would be quite complicated if they were expressed in words. But algebra is not just a convenient shorthand: it leads to ways of rethinking and re-organizing relationships. By following mathematical rules, symbolic expressions can be *manipulated*—changed in form—to reveal new relationships or to solve problems.

To round off the block, *Unit 9* focuses on music as a source of mathematical ideas about representation and relationships. The unit looks at the mathematics embedded in the notions of musical scales, pitch, intervals and different temperaments (tunings). Do not worry, you need no special musical knowledge to enjoy and learn from this unit.

Introduction

This unit is all about representations and relationships. You may not always be conscious of it, but representations are all around you—on advertisements, in newspapers, at the supermarket, even in your study notes! A key component of representations is the 'language' used to form the 'picture'. Many representations use graphic images of different sorts to represent the story. Maps use a particular language of special symbols, and different maps use different symbols to tell their stories. To read and interpret a map, just as in reading any other representation, you need to be able to make sense of the symbols and understand what they are trying to say. And, as you have already seen, mathematics uses its own special language—numbers, symbols, diagrams—to express different ideas and relationships.

You have already seen in Block A how series of numbers can be used to demonstrate a relationship—for example, between cost of living and income; and how numbers can also be used to create an image—a graph or pie chart. Using numbers in this way helps us to make sense of them—that is, create an interpretation.

The first part of this unit considers how maps are made to represent the world and the ways they can be read and interpreted from an understanding of the symbols they contain. Part of this interpretation requires a knowledge of particular mathematical skills and techniques and, although you will be introduced to them through maps, they are, in fact, skills and techniques you will use many times in this course, in different contexts.

The second part of the unit takes some of the ideas introduced through maps and looks at how they can be represented in another form—through graphical representation. For example, you can describe a land surface as 'steep' or 'flat' or 'rolling', but you can also estimate *how* steep it is by looking at the relationship between distance and height. Such a relationship can be represented as another type of picture—a profile or graph. Graphs are around us all the time and are not only used to tell about the steepness of the land. You can see them in hospitals, banks, and in the media. You saw them used in Block A, where they provided a convenient way of displaying ways in which two things are connected: temperature and days, interest rates and months, spending power and years.

Throughout this unit you are asked to think about how representations come about—are they 'true'? Do they incorporate or reflect a bias or a particular viewpoint? A key theme running through the unit is always looking out, in any representation, for what is stressed and what is ignored. To create a representation you need data and the important process of data collection starts by the selection of particular pieces of information and the exclusion of others.

In Block A, you completed a number of activities involving planning and monitoring your work schedule for a unit. In this block, try to record your schedule for study before you begin to work on each unit, but now also include one area of your study that you particularly want to improve. For example, you may want to concentrate on developing your calculator skills, or to try a different way of making notes, or to set aside more time to complete the activities, or to try different strategies when you get 'stuck'. As well as writing down this one area where you want to make improvements, try to say how you intend to go about it and how you will know that you have been successful. There will be an activity in each unit of this block to help you do this.

Activity 1 *Planning to make improvements*

Plan your study for *Unit 6*, using the study guide diagram. As well as taking account of the different components you need to use, think about how you are going to complete the assessment for this unit. From your experience of the assessment questions for Block A, decide when you will look at the assessment material and when you will begin preparing it. Do you intend to do anything differently? For instance, will you make notes for the relevant TMA question while you are studying the unit or after you have completed the unit? How do you intend to use your tutor's comments? Is there any other information you need to make improvements?

Record your schedule in a way that you find useful (as a representation!).

Think about one area of your study where you want to make changes. How do you intend to go about doing this and how will you know when you have achieved success?

A printed sheet for planning and monitoring your study is provided but you may prefer to develop your own.

1 Picturing the world

Aims The main aim of this section is to introduce you to the idea that all maps are particular representations, and in using them you need to be aware of features that are stressed and those that are ignored. ◇

Next time you look at a world map, take a close look at the Greenwich meridian line – an imaginary boundary that divides the world into east and west hemispheres, and against which reference point most of the world sets its clocks. The Greenwich or Prime Meridian passes through the birthplace of modern navigation (the Greenwich Observatory in London) and demonstrates vividly the human mind's ability to think abstractly and symbolically. Because people could envisage an imaginary grid of lines overlaying the surface of the globe, certain mathematical skills, that now allow the calculation of any position at any time upon the Earth's surface, were developed.

Success in this task, which was achieved in the sixteenth century, meant prosperity for nations whose merchant fleets could then successfully navigate the seas. Mathematics, which involves representation and thinking symbolically, is an important foundation of navigation and thus was an essential component of England's economic success at that time.

However, different people use maps in different ways and for different purposes. Sometimes these purposes diverge from those for which the map was created. For example, an old map created for navigational purposes may today be beautifully framed and hung on a wall as a work of art. An Australian aboriginal drawing may be used as a support for finding one's way, for telling cultural stories of how the landscape features came to be that way, as a visual record to help with recounting the history of a particular group, and much more.

Much of this section provides background information which you are not expected to remember. You begin your study of this unit, by looking at a variety of maps to consider their features, design and possible uses. By working with a small number of different maps, you are encouraged to clarify what you already know about making and reading maps, and begin to consider how to interpret these representations by thinking about what is included, omitted and distorted: in other words, what is stressed and what is ignored.

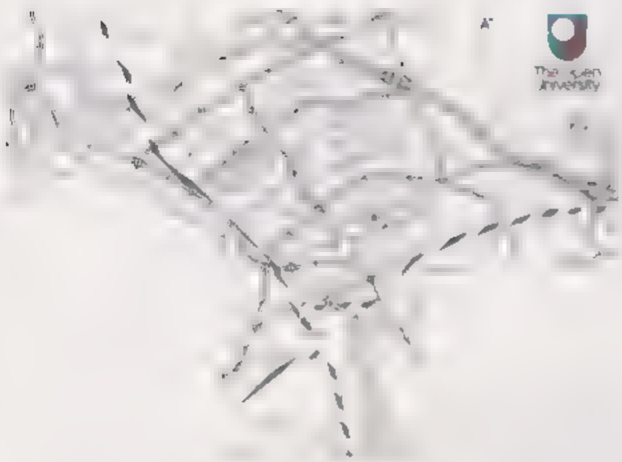
You may find it useful to refer to the outcomes for this section at an early stage to help you identify the main learning outcomes (on page 34).

Activity 2 *What can you see in a map?*

Look at each of the maps in Figure 1. For each one, make a note of what you notice about it; what it seems to tell you; how you know this; and then ask yourself for what purpose you might use that map.

At the end of the unit, you will be asked to produce your own commentary about maps, so make notes, including a reference to where you were in the unit.

A printed response sheet is provided for this activity.



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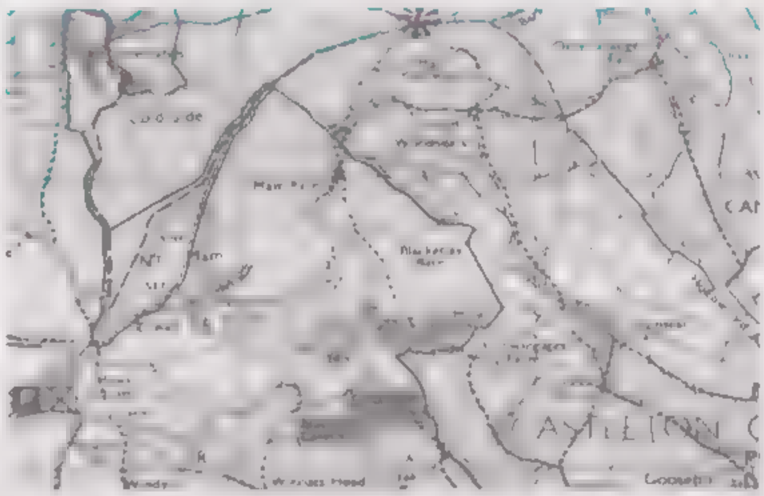

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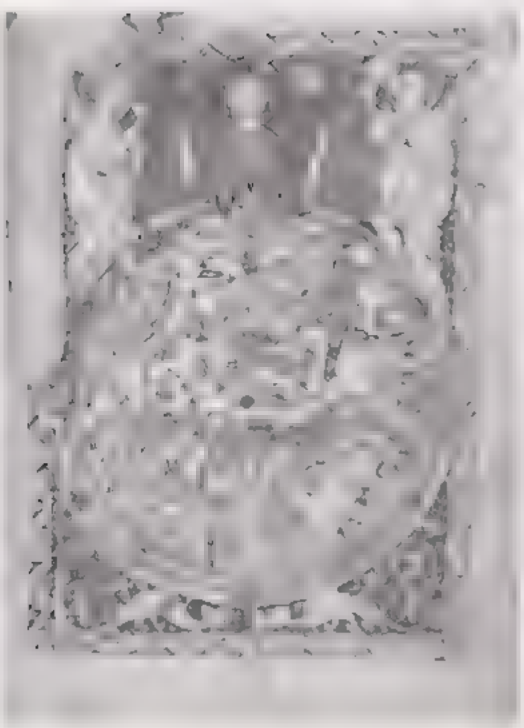

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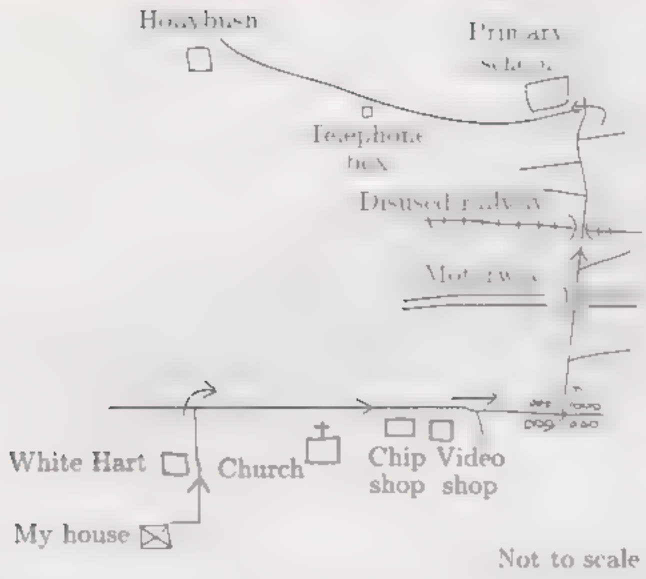

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Figure 1 Maps for use with Activity 2

Maps allow an interpretation to be made of aspects of particular areas of the Earth's surface—and they provide an insight into the contrasting way in which a region is seen by different groups of people. For example, the way in which area is measured can differ and even the views of distance may not be the same. In a land conflict case in Honduran Mosquito, Central America, in 1992, between the indigenous people and the government, the farmers' concepts of distance and space were questioned. However, the different perceptions came not from a lack of knowledge about distance and space, but from a different kind of knowledge.

This indigenous people's ideas of space and measurement of distance was in terms of their practical relationship with the area concerned. Distances, for instance, were measured in terms of 'how many hours it takes to walk to these places'. The government officers, by contrast, thought in terms of units of measurement—hectares and kilometres—measures which ignore variation in terrain. While the indigenous people's main concern was the integration of the land into the rhythm of their daily lives, the concerns of government needed abstract units of measurement which would enable them to compare the area for 'development potential' with other areas elsewhere. Thus, the interpretation and hence significance and representation of the same area from the two groups were quite different.

By thinking about how you see and represent parts of your own world, you may already be formulating some statements about maps and mathematics, such as:

> I hadn't seen such a variety of maps and thought about the different uses of them.
>
> I hadn't realized that many maps are designed and produced for a specific purpose.

> I hadn't considered whether maps are objective or if they contain bias—I hadn't thought about the idea of stressing and ignoring. I just thought of them as pictures.
>
> I had never thought about mathematics being involved in the making of maps.

Add any questions or statements you have to the list as you work through the unit. In considering the different maps in Figure 1, you were accessing your own knowledge and experience about maps and representations. You may feel that your goal is to move on, and not look at what you already know. But if you are to make sense of new information being presented—if you are to understand it solidly—you need to relate it to your existing knowledge.

Understanding 'solidly' can be thought of as a two-way interchange between pre-existing knowledge and new information. Pre-existing knowledge is used to help interpret the new material and, in turn, the new material yields information that may be used to modify pre-existing ideas and beliefs, sometimes in major ways. Take a moment or two to think about this in relation to your study of Block A. Thinking explicitly about what you already know about maps should help you to integrate new knowledge and develop your understanding.

One general idea arising from the maps presented in Figure 1 is that maps are a means of representation and every individual map embodies *both* a particular way of seeing and understanding, and a particular interpretation of the place it is depicting. To an extent maps are 'social products'. Their design—for instance, what they stress and what they ignore—reflects different experiences, priorities and interpretations. A map you may have drawn to show someone how to get from their house to yours is doing just that—expressing the priority of how to reach one place from another. Yet another aspect that you will see in later examples is that putting things on to a map can signal power, or give access to power. In just the same way, omitting things can mean lack of power. Maps used, for example, for defining territory can give powerful messages in the language used to name places, rivers, lagoons, mountains and other physical landmarks.

Within the UK, whether a map calls the town 'Derry' or 'Londonderry' can convey a great deal of information inside Northern Ireland.

Maps, then, are human creations. They express particular interpretations of the world, and they affect how people understand that world, and how people see themselves in relation to others. In making use of maps, you need to be aware of this.

1.1 What makes a map useful?

Making maps is a complex business and for a map to be useful to a wide variety of people means decisions have to be made. Maps are a particular way of representing certain aspects of the world. Like other forms of

representation, they are a means of locating yourselves in, and finding your way about in, a larger context. They are a way of making sense of complexity and of helping to get a grip on where you are.

Maps are always the product of particular societies and particular groups of people. They reflect specific ways of thinking; each is designed to serve a particular purpose. At its simplest, no map can show everything —to do so it would have to be as big as, indeed absolutely the same as, the world it represents. So things have to be omitted. And that process of selection can tell you something about the perceptions and intentions of the map-makers who produced the map.

The process of deciding on map data starts by the selection of particular pieces of information and the exclusion of others. For example, the Ordnance Survey map extract you have as part of this unit excludes information on the rainfall and details about the population of the area—their occupations, heights, weights, medical history, and so on. In fact, nearly *all* information is excluded. By selecting only part of the real world to represent, the map designer ignores many, many things.

Historical note

Maps are not the only representations where you can find selection of information. Another interesting example can be found in art. The art historian, Ernst Gombrich has commented: 'so complex is the information that reaches us from the visible world that no picture will ever embody it all', adding '... the correct portrait, like the useful map, is an end product on a long road through schema and connection. It is not a faithful record of a visual experience but the faithful construction of a relational model. ... the form of a representation cannot be divorced from its purpose and the requirements of a society in which the given visual language gains currency.'

E. H. Gombrich (1962) *Art and Illusion* (Phaidon Press, Oxford) p. 78

For map-makers, it is the intended purpose of the map which determines the objects to be included. For example, a small scale map in a railway passenger timetable would not normally include lines used solely for freight. Once the selection of objects or relationships has been made to the satisfaction of the map-maker, sometimes acting together with the map-user, concentration will focus on these often to the exclusion of everything else—you will see a good example later in this section when you consider network maps.

Maps also reflect ways of thinking. They provide an insight into how the people who produced them imagined the world and how they placed themselves within it.

Consider the variety of maps that you have already encountered, and the potential for bias—both towards features that are stressed and away from those that are ignored.

On bias

Bias is a commonly-used word. You hear about politically-biased commentary on TV and radio, a judge being biased, coins or dice being biased, and so on. The origin of the term is in loading containers unevenly resulting in an oblique motion. Bias can be towards or away from something, and material cut 'on the bias' means a cut on the slant rather than straight across the grain of the fabric.

The term 'unconscious bias' indicates that bias need not be carried out intentionally or deliberately in order to produce a biased effect. So when the unit speaks of 'biases' of map-makers, it is not necessarily assuming deliberate intent to skew or deceive.

So what makes a map useful? Clearly, a map usually needs to portray spatial information efficiently on a flat surface, to be useful to map-users who may be engaged in a wide variety of activities—from recreation, legislation and decision-making to navigation, education and decoration. To achieve this goal, particular conventions are used.

Conventions—styles of presentation—may include some kind of reference title, a scale, orientation information (sometimes shown as a North arrow), and a key for any symbols used. In future, when you see maps, check whether they do contain this information. You might like to look back at the maps shown in Figure 1 and the OS map extract to see if these conventions have been used. Before moving on to look at map projections and maps produced for particular purposes, one of these conventions—the orientation of a map—is discussed. The ideas of scale and symbols are covered in later sections.

The orientation of a map

When people open up a map, they often take for granted that the map is designed so that north is at the top of the page and south is at the bottom; in other words, that it is orientated north–south. Today, most maps obey this convention. But have you ever wondered why, who decided and when the decision was made?

There is no 'right' way up for a map. You may find that when you are using a map to follow a route, you actually turn the map round so its orientation is the same as your own. It can then be easier to compare the map with the reality it is supposed to represent. However, if you had a set of maps where each had its own orientation, then combining information or comparing ideas from them would become complicated.

The very word 'orientation' reveals that some of the earliest maps pointed east (*oriens* in Latin), probably because Paradise (or its equivalent) was for centuries believed by some to lie in the easternmost part of Asia. It is shown in that position on the Psalter world map of about 1260 (Figure 2), which is now believed to be a spiritual representation. Physical geography

is more or less distorted to accommodate the map's principal purpose: to provide a framework within which the map's creators' Christian view of human existence, faith, history and knowledge could be expressed in pictures.

Figure 2 Psalter world map

The north orientation of most maps is no more than a convention which may have arisen from early work involving the development of compasses and their behaviour relating to the Earth's magnetism.

Historical note

Around 2500 years ago, the Greeks discovered the magnetic properties of lodestone—the mineral called *magnetite*, which is one of the ores of iron. By the first century AD the Chinese had used it to construct the first compass in the form of a lodestone spoon balanced on a smooth plate (Figure 3). Travellers of the fourteenth century brought news of this discovery back to Europe, where the device was adopted as a navigational aid and played a major part in the voyages of such explorers as Columbus and Magellan.

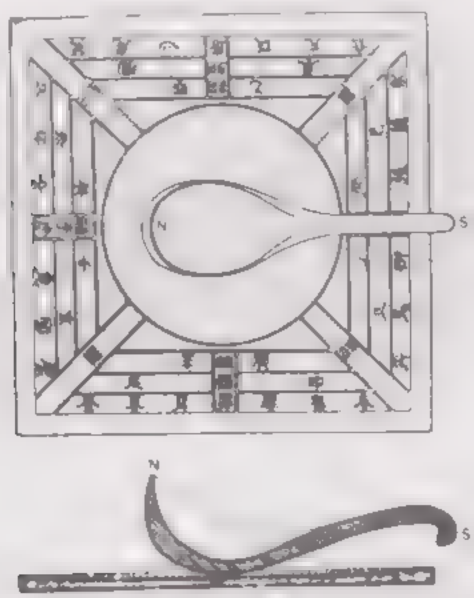


Figure 3 Model lodestone spoon and bronze plate reconstructed during the 1940s from an ancient Chinese pattern. This is the earliest form of the magnetic compass. The handle of the spoon points approximately south

It was William Gilbert, physician to Queen Elizabeth I, who explained in his book *De Magnete*, published in 1600, how the magnetic compass works. He offered the thought that 'the whole Earth is a big magnet', whose field acts on the small magnet of the compass needle to align it in the north–south direction (Figure 4).

This has been further confirmed by recent space exploration. Mariner 10 showed that Mercury has a significant magnetic field, with north and south poles aligned like the Earth's with the planet's rotational axis, but whose strength is only about one-hundredth of the intensity. Mars and Venus both lack significant magnetic fields. The Moon has none. For many on Earth, magnetism and navigation are intricately linked ideas.

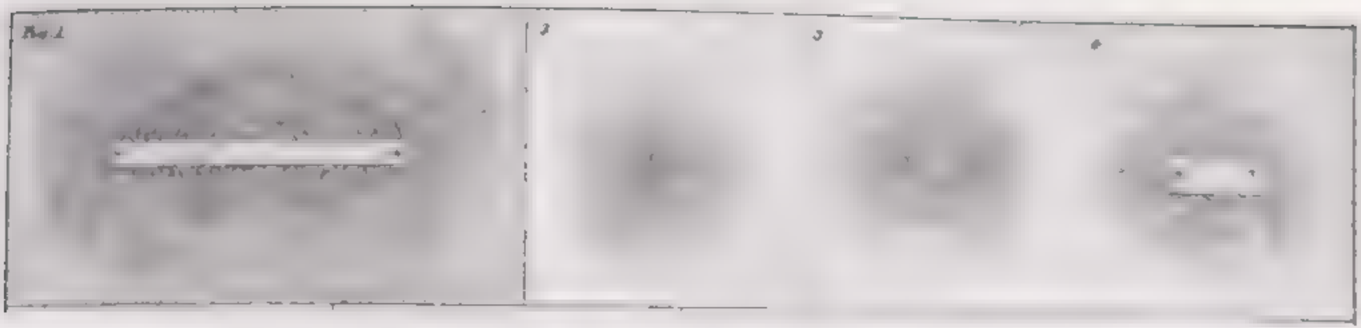


Figure 4 The magnetic field of a bar magnet shown as a pattern of iron filings on paper. The filings are aligned parallel to the magnetic field lines

Today, many people are unsettled by anything other than a north-orientated map—demonstrating the strengths of present convention! However, as recently as 1979, Stuart McArthur of Melbourne, Australia created his own map of the world entitled 'McArthur's universal corrective map of the world'.

▶ What point do you think McArthur might be trying to make through this map?

Figure 5 Map of the world, after McArthur

The traditions of map-making reflect the customs and culture of which the map is part. Other cultures may view the world differently and so a different convention may reflect the priorities of their culture. In McArthur's map, you possibly noted that Australia is centrally located and south is at the top of the page: the orientation is south–north. This map reflects the increasing importance which Australia now gives to being in the southern hemisphere and on the Pacific rim, rather than looking so much towards its colonial past and Britain in the northern hemisphere.

Finally, have a look now at the OS map extract and see if you can see the orientation it takes and how it is recorded. Keeping orientation constant is useful so that maps can be more easily understood, interpreted and used for comparison.

1.2 Squaring the circle

The graphical and mathematical problems of representing the (roughly spherical) surface of the Earth as a flat map have taxed the ingenuity of map-makers from ancient times onwards. It was accepted that all maps were compromises, capable of portraying one or more of the Earth's features, yet unable to show accurately on the same flat surface, the four essential elements of true *shape*, equal *area*, accurate *distance* and consistent *orientation*. Only globes were capable of reflecting such features of reality, but these miniature models of the Earth had disadvantages. Even the largest had a very small scale and therefore could show relatively little detail. Globes were also costly to make, difficult to store, heavy and impractical to carry around. The obvious solution to the problem is to substitute a flat map for a bulky sphere.

Mapping the surface of a globe on to flat paper without tearing or distorting the paper has challenged many people. Current mappings of the world are framed within the viewpoint of scientific studies, and they can draw both on the accumulated knowledge of centuries and on the high-technological processes of recent years. And yet, even within a scientific framework, there are still differences and debates about how the world can best be represented. Decisions still depend on for what purposes (and for whose purposes) the map is being made. This has created a whole area of mathematical study related to map projections.

The main point of these studies and debates concerns the technical question of projections—of how to resolve the difficulty of representing the nearly spherical surface of the planet on a flat surface. Some kind of choice always has to be made—certain features will unavoidably be stressed and others ignored. Moreover, it is not only a technical question. Different map projections reinforce different, and particular, views of the world.

Next time you peel an orange, cut the peel off in sections and arrange it on a flat surface in the same way as shown in Figure 6.

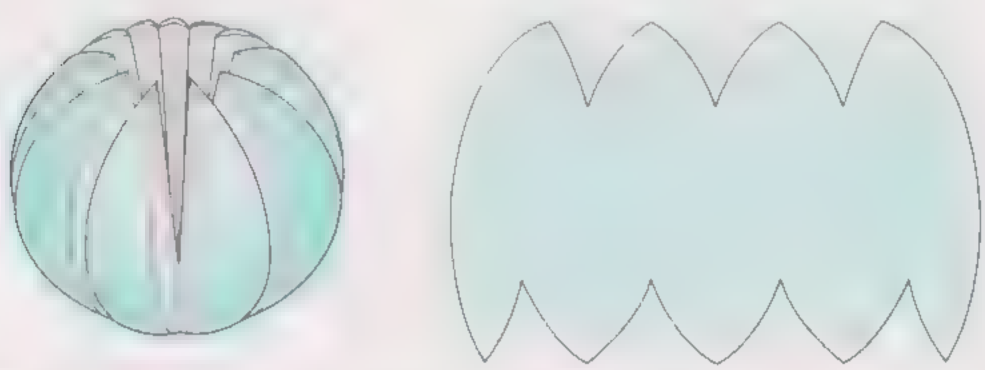


Figure 6 Orange peel arranged on a flat surface—a projection

▶ What do you see?

You have created a particular projection of a broadly spherical object on to a flat surface and you should be able to identify where accuracy is retained, and where distortion is created. In designing and producing maps, the projection which map makers select for a map will be influenced by a variety of interests: their own, their customers, their employers. The result, over the centuries, has been a multitude of images of the Earth, all of them necessarily containing some truth and some distortion, serving some ends but prejudicing others.

You might find it helpful to think about this problem of 'unwrapping' the globe on to flat paper by adopting one of three basic approaches—'stretching', 'squashing' or 'snipping'. You can keep shapes correct, but harm the scale; or you can keep scale correct, but harm the shape. A compromise can offer less extreme distortion of scale and shape, but it is not then accurate in either respect. 'Snipping' can keep size and scale almost correct by putting the distortions into the cuts. These cuts are normally made in the oceans.

▶ Which approach have you adopted for your orange peel projection?

You possibly used all three. All these approaches are in use, as you will see in the following examples, and they are all termed 'cylindrical' map projections (Figure 7).

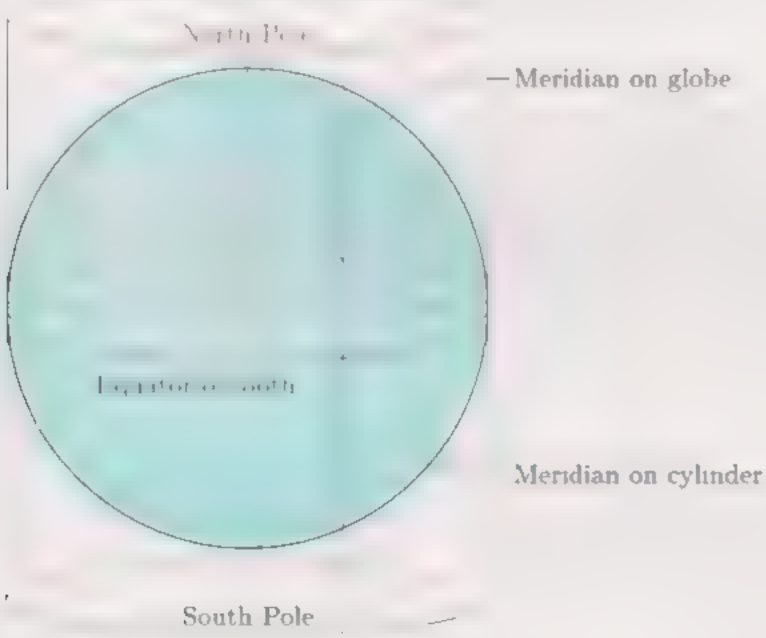


Figure 7 A cylindrical projection

You can visualize them by imagining a cylinder wrapped round the globe and touching it all around the Equator. When the lines of latitude (lines running around the globe and parallel to the Equator, sometimes called *parallels*) and longitude (lines running from pole to pole, sometimes called *meridians*) are transferred onto the cylinder, it can then be opened out flat to form a map sheet. Projections can be of immense value in providing new ways of looking at the world, and are becoming more so as computers offer greater flexibility in creating new ones. But projections affect the way people perceive the planet and so they all have emotive and political undertones. And like the different maps you have already considered, each projection will stress particular features and ignore others.

During the sixteenth century, a Flemish map-maker, mathematician and instrument maker named Gerard Mercator, developed his world projection with the prime purpose of facilitating navigation. At that time, what was needed were accurate world maps and charts. His projection involved drawing the meridians (lines of longitude) as straight lines an equal distance from each other and at right angles to the parallels (lines of latitude), creating a grid called a *graticule* (Figure 8). This construction allowed any constant compass bearing to be plotted as a straight line on the map. The Mercator projection is the best known of the 'stretch' maps, and is still used by navigators. Land shapes are excellent but countries like Greenland, Canada and Russia appear far bigger than they would on a globe.

▶ From Figure 8, what is the visual effect of Mercator's projection upon the apparent size of the land masses as you move from the Equator towards the polar regions?

Preserving constant compass directions means sacrificing the true area and proportions of the Earth's land masses. This results in massive enlarging as you move towards the poles—everywhere except at the Equator there is east-to-west stretching; the only line of latitude where distances remain true is the Equator. Greenland, which would fit into South America over eight times, appears the larger of the two. The actual poles cannot be projected at all.

Aware of these limitations, other map-makers have employed different models for their projections of the world's surface. Arno Peters used the principle of 'equal area', which represents equal areas on the ground by equal areas on the map. This projection gives the reader the correct impression of relative land masses, but distorts angles and directions and so the true shape of coastal outlines is distorted (Figure 9).

Figure 8 Mercator's projection

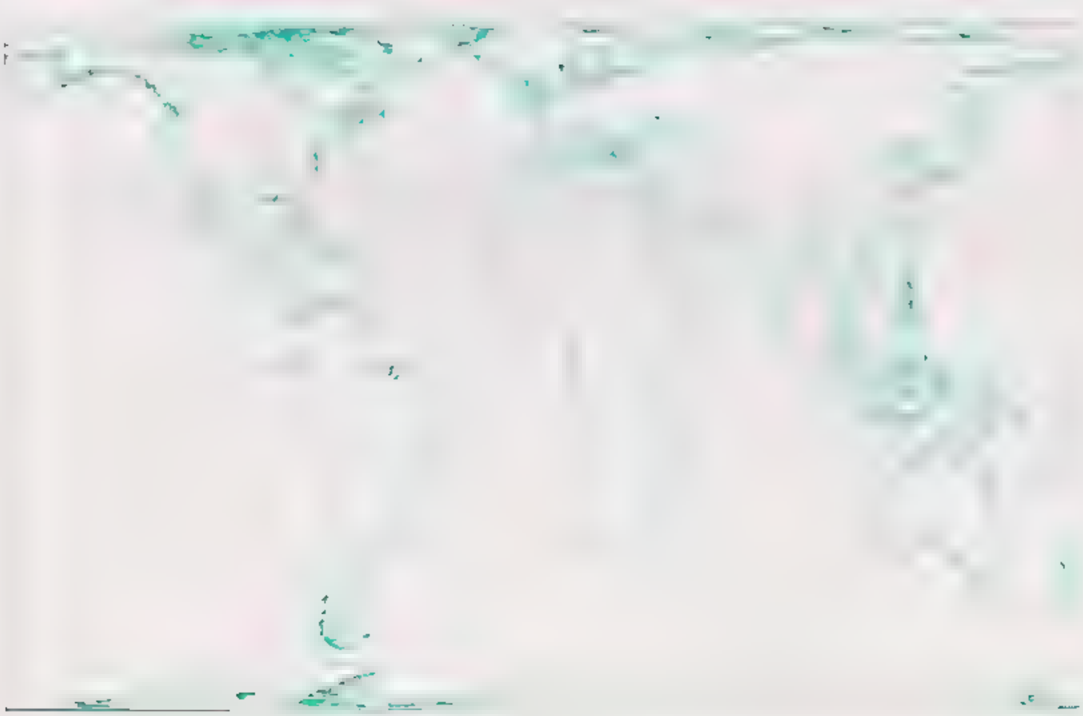


Figure 9 An equal-area projection of the world

In 1963, the American cartographer Arthur Robinson embodied the principles of both projections—equal compass direction and equal area—to produce his projection of the world (Figure 10).

Figure 10 Robinson's projection

➤ Look back at the three projections of the world (Figures 8, 9 and 10). Can you suggest why the Robinson projection has been popular in the USA only since the ending of the Cold War?

This provides an example of the political implications of map projections—the signalling of power in the way maps are used to redefine the way the world is perceived rather than merely, neutrally, to represent it. Until recently, the Mercator projection was widely used in American magazines, since it gave readers the impression of the vastness of the USSR in comparison with the USA and thus the apparent size of the Soviet 'threat' to the USA. Since the ending of the Cold War, the Robinson projection is now widely used in American magazines, producing an image of America's new 'ally' which is altogether smaller relative to the USA, and consequently able to be perceived as less menacing. In contrast, Mercator's 'world' projection is increasingly labelled 'Eurocentric'.

Maps have an important role to play in current affairs. Sometimes, as in land claims disputes, the map is the focus of the disagreement. But you need always to remember the context of who drew the map and under what circumstances.

Activity 3 Transforming 3D to 2D

List the main elements to consider when designing a representation of a globe, a three-dimensional (3D) object, on a flat piece of paper, with its two dimensions (2D).

What other examples can you think of where a 3D-object is transformed to a 2D-image?

Study the reader article 'Blindly into the ditch'. Explain, in your own words, how this 'error' has come about. What lesson can you learn from this and have the drawn figures in this section managed to avoid falling into a similar 'ditch'?

Of the properties you have identified in Activity 3, it is not possible to preserve all of them in creating a 2D image from a 3D-situation. Decisions have to be made.

Activity 4 Making choices and justifying decisions

Imagine you are involved in making each of the following maps. State which principles and/or conventions you would consider most important to implement, giving your reasons for including or excluding each.

(a) A sketch map for a friend to find her way from the nearest bus stop, when she comes to visit you.

(b) A map for an airline pilot to navigate with on a flight over the North Pole.

(c) An Ordnance Survey map of a mountainous area.

(d) A world map showing the historical journeys of European explorers to Australia.

In conclusion, no world map is neutral and none are totally true. For them to be useful, it is important to be aware of the limitations of any map, whether of the whole world or of just a particular place.

1.3 Maps for particular purposes

The problems posed for map-makers continue to create particular challenges, and it is all too easy to look at a map and see the finished product which reveals little of the work involved in creating it.

An early stage in the design and production of any map is specifying its intended purpose, as was the case with Mercator's sixteenth-century navigation maps. However, although maps may be made for specific

purposes, there is no guarantee that they will always be used in the way intended. For example, the one inch to one mile Ordnance Survey map of Britain is an example of a topographic map (see below) made initially for military purposes but used extensively by non-military personnel.

Part of designing a map involves the map-maker taking information about the world and transforming it in a number of ways before it is presented in idealized form as a visual, symbolic model. This information is coded in symbolic form—you could even go as far as saying that the entire map is a complex, inter-related symbol. The map-users then take over, to translate the aspects of the map they are interested in. Symbols comprise the language of map-making and indeed of mathematics.

As you work through the next series of case studies, try to identify the particular problems faced by the map-makers and think about the way they were overcome. Activity 5 asks you particularly to look out for any mathematical solutions to the problems.

Activity 5 *Identifying problems and solutions*

As you work through each of the case studies, identify the mathematical technique that the map-makers have used to solve the particular problem. On the printed sheet, make a note of the technique, particular features relating to it and an example of how it is used. Continue to add to your list of techniques and examples as you work through the unit.

The aim is to create a listing of techniques that can be used in different situations.

At this stage, you may also find it useful to start using the Handbook sheet for terms and definitions.

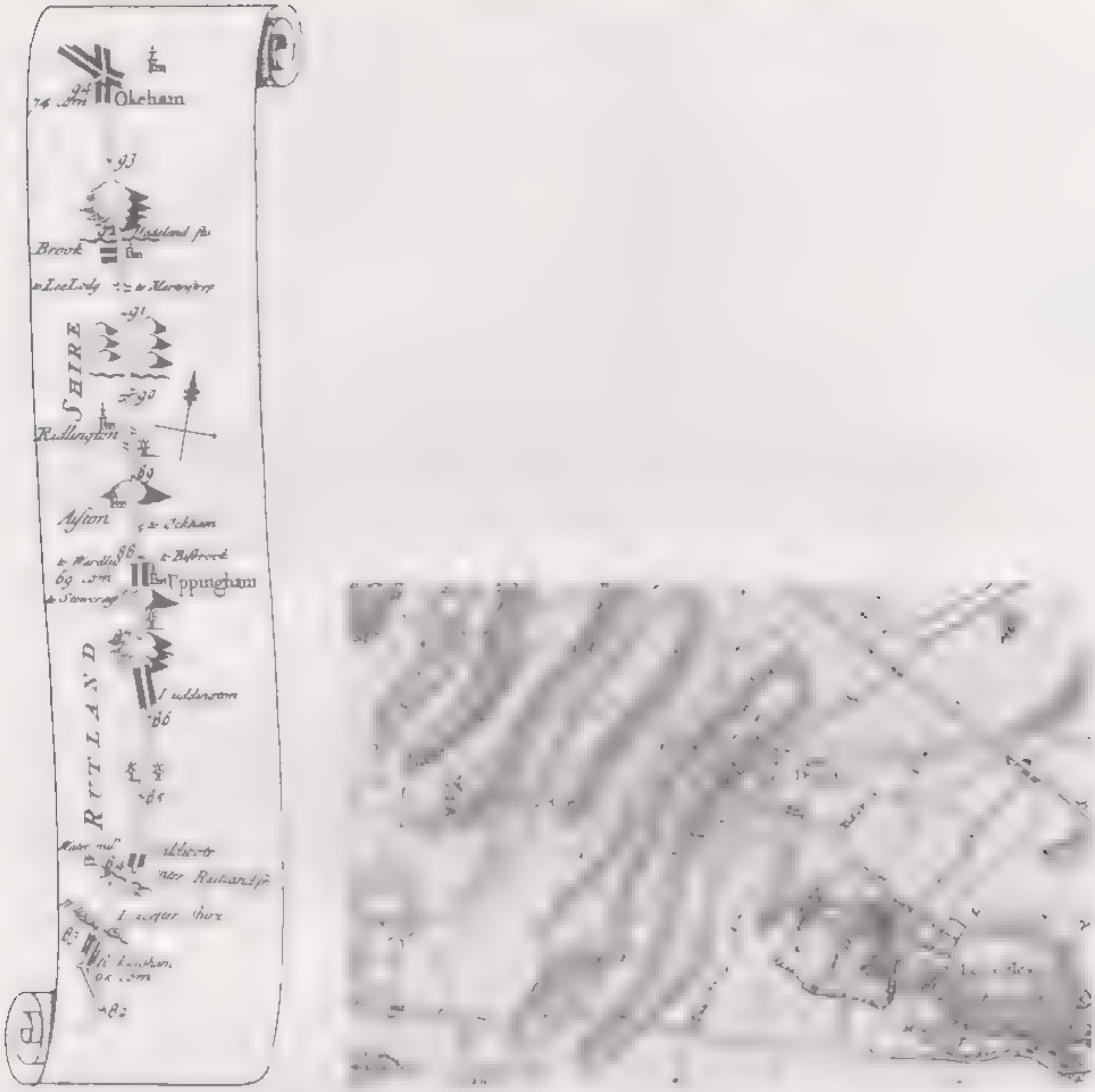
Case study 1 The making of a topographical map

Recall from *Unit 1*, Alan Bishop's claim that 'locating' was one of the fundamental activities that all peoples engaged in — an activity he saw as intrinsically mathematical.

Cartography, the science and art of the drawing of maps, is one of the oldest human activities. Some of the earliest known maps are preserved on ancient clay tablets from Babylonian times and, it seems from very early in human history, there was a need to represent the relative positions of known places. Such maps needed to show the essential geographical and landscape features; good maps did not generally become available in Britain until about the end of the eighteenth century.

The geographical features of a land surface are called its *topography*, and maps showing these details are called *topographic maps*. Hills and valleys, undulations of the land surface, are known as the *relief*. One of the problems that early map makers faced was how to portray the relief of the land surface on a flat piece of paper.

On the earliest maps, hills were shown in a very diagrammatic way as little 'cones' dotted about the map to represent where the top of the high ground lay (Figure 11(a)). Later, map-makers used the technique of *hachuring*, in which the side of the hill is shaded to show slopes (Figure 11(b)).



(a) (b)

Figure 11 [illegible] from London to Oakham in Rutland drawn by John [illegible] (b) Detail from the Cambridge sheet of the First Edition Ordnance Survey [illegible] England and Wales (1836). Scale 1 inch to 1 mile

During the last decade of the eighteenth century, several national agencies were set up as a result of pressure exerted by various people who recognized the value of some centralized control. In 1791, the Board of Ordnance was formed under the control of the British Army since the need was clear for more accurate and systematic knowledge of the nature of the landscape, especially for defence in the state of war between Britain and France. Thus all the early surveyors were serving officers of the British Army and, until the mid-1970s, the post of Director-General of the Ordnance Survey was held by a senior army officer.

The word ordnance refers specifically to military weapons and more generally to military stores.

The method of showing relief by hachuring persisted well into the nineteenth century. However, the work of the Board of Ordnance introduced a more systematic approach: a detailed survey of the country was carried out and heights of prominent hills were related to sea-level. The most important step of all in topographic mapping soon followed: the representation of relief on maps by contours drawn at regular height intervals. Each contour on the map represents the intersection with the relief of an imaginary horizontal plane which is a particular height above mean sea-level. Mean sea-level is used as the reference plane, and is known as the Ordnance Datum (OD). Later advances in UK cartography included the standardization of map scales and the establishment of the National Grid system of reference which enables particular features to be located easily. (You will learn more about these ideas later in the unit.)

'Mean' here refers to the arithmetic mean discussed in *Unit 2*. Recall 'datum' is the singular of 'data', meaning a single item of numerical data.

The idea behind contours is not unique to the representation of height on the land surface. Lines joining places of equal numerical value with respect to a given climatic or other variable are called *isograms*. Examples in common use include isobars which join places of the same atmospheric pressure; isotherms which are lines of equal temperature; and isohyets which link places of equal precipitation (contour lines of equal rainfall). You may be familiar with isobars on weather maps.

The Greek prefix 'iso' means 'equal'.

Case study 2 Mapping diseases

In *Unit 4*, you studied a number of examples of maps that were created to show the spread of disease. In America during the 1790s, the spread of yellow fever was recorded by using 'spot-maps' giving the location of infected households. You may also remember from *Unit 4* that in 1855 John Snow's essay 'On the mode and communication of Cholera' included a map which told the story of his research into a cholera epidemic. It was a map of Soho (London) on to which he recorded cases of cholera. The mapping revealed a high concentration of the disease in the region of the Broad Street water pump. This may have helped support his theory that cholera was passed on through the drinking water and as a consequence the water pump was chained up and residents were forced to seek alternative supplies. This in turn helped control the epidemic.

The mapping of the spread of disease can enable forward planning in terms of treatment, care, prevention and education. The purpose influences the way in which the map is drawn or even whether a map is used at all. Peter Gould's book, *The Slow Plague: a geography of the AIDS pandemic*, asks the question: 'Why in the first dozen years of the AIDS pandemic has the map virtually disappeared from use?'

Peter Gould is Evan Pugh Professor of Geography at Pennsylvania State University in the USA.

Activity 6 *Patterns of disease*

Study the reader article 'Mapping the AIDS pandemic' by Peter Gould. What information has been collected? Can you suggest reasons for a reluctance to map the spread of AIDS?

Remember to include an entry on your sheet for this case study.

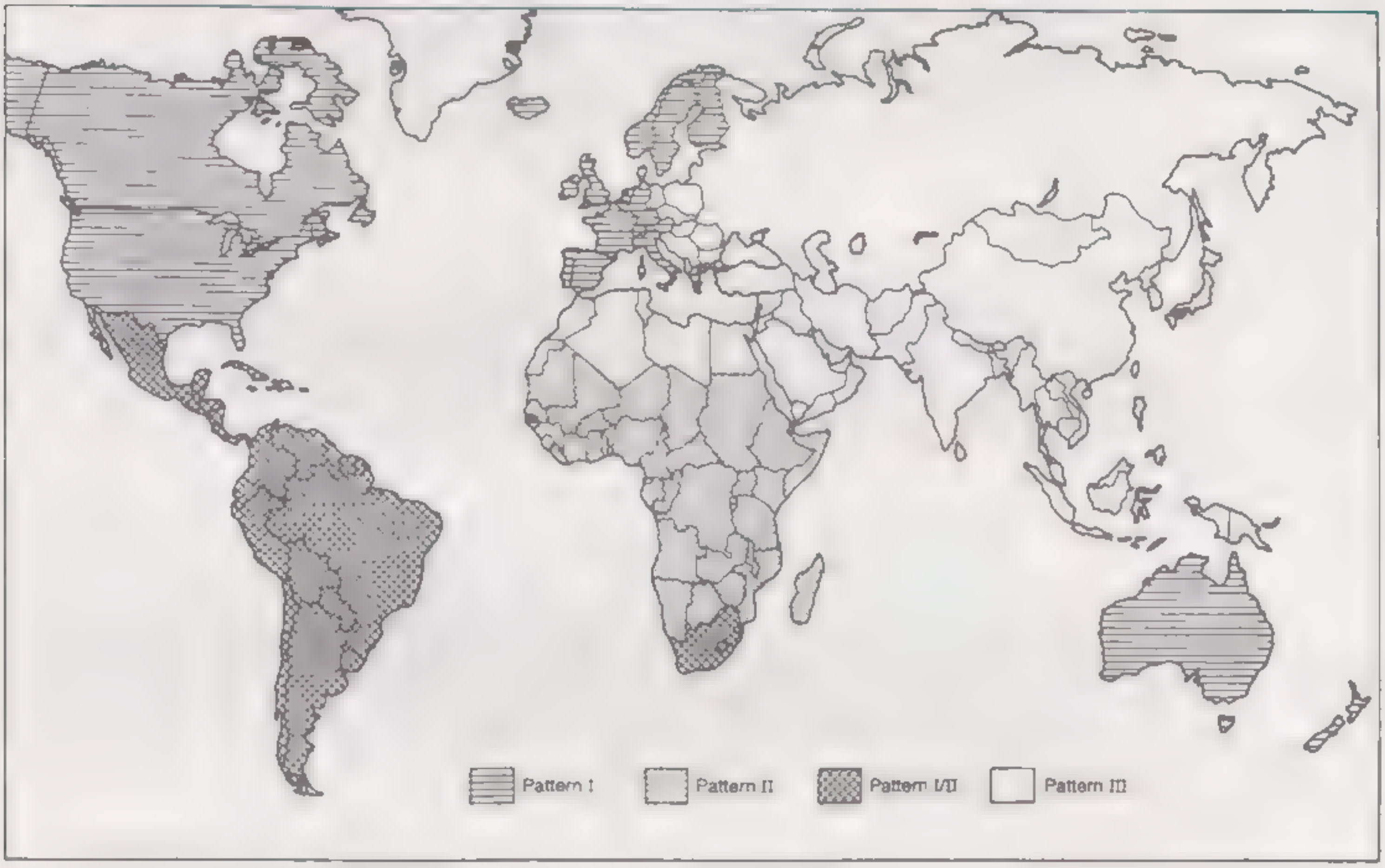


Figure 12 Map of AIDS transmission, 1991 (later versions of this map exist)

Case study 3 Tactile maps

Tactile mapping can help visually-impaired people acquire a sense of spatial relationships that sighted people take for granted.

Tactile maps have raised lines and symbols, so that you can *feel* rather than see the map. The notion of tactile maps may be a new idea to many people: it is a very useful but simple idea. Maps for blind and partially-sighted people have been produced to provide them with greater freedom to travel or explore and be informed about new areas: for example, the layout of a railway station, airport terminal building, museum gallery or the route for a walk. Figure 13 shows a picture of a *tactile map* of a canal walk together with a sketch map of the same locality

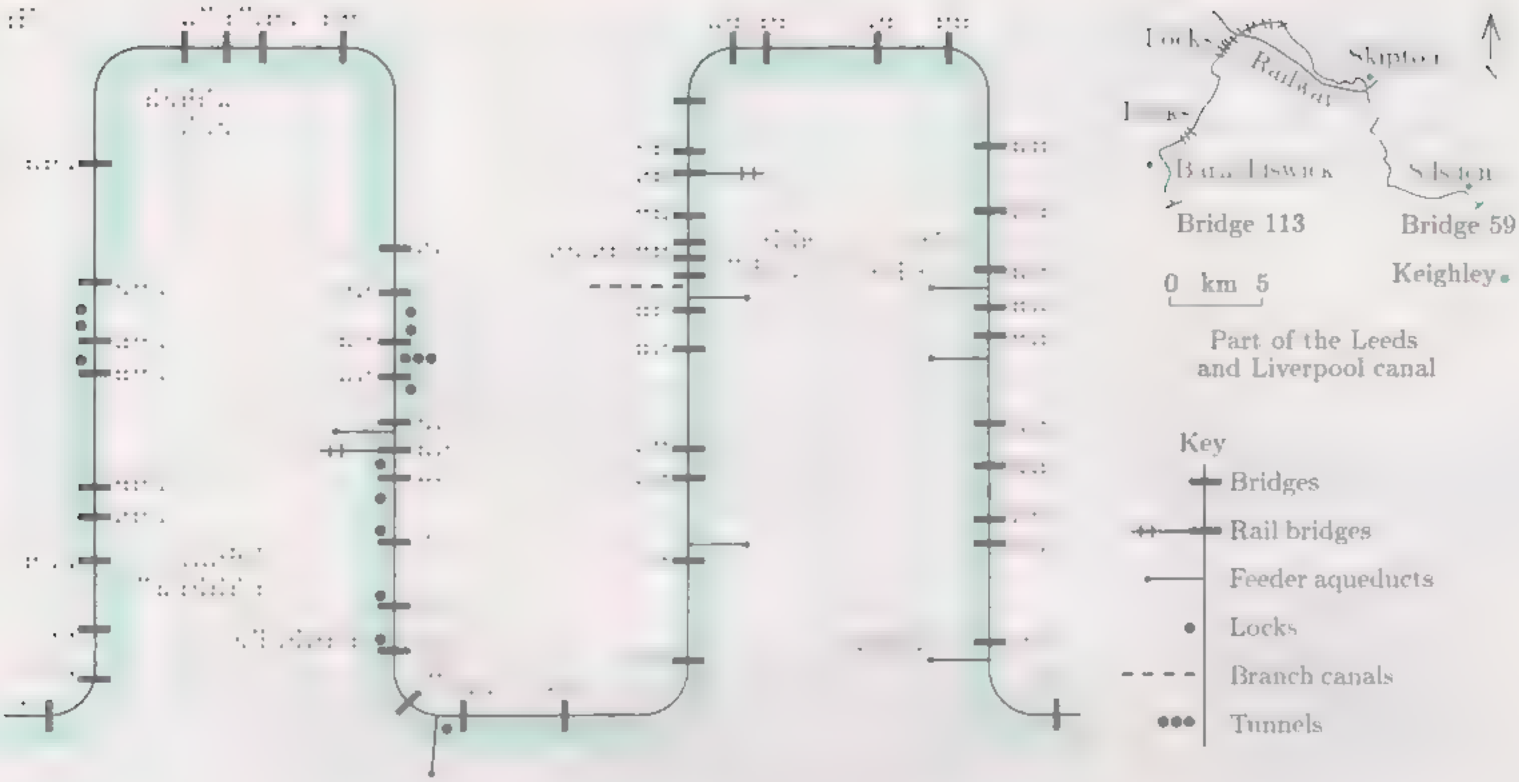


Figure 13 Diagram of a tactile map and sketch map

Activity 7 *Principles of a tactile map*

Study the map and associated sketch of the region, then write down your observations on the principles involved in producing a tactile map. What features would you include in constructing a tactile map? Consider the content of the map and the needs of the intended user.

Using these ideas as a basis perhaps, think about the place where you live and design a simple tactile map which would help a blind or partially-sighted person to navigate themselves around part of your home or your locality with reasonable freedom and independence.

Case study 4 Network maps

Networks are examples of representation that help to describe relationships by identifying patterns from a mass of detail. Particular users—for example, distribution agencies—need to be able to identify and monitor particular routes. A conventional road atlas may present too much detail for this purpose; what would be more useful would be to identify some kind of pattern for the distribution so that progress could be monitored, areas compared, and future plans drawn up. Network maps are often constructed for these purposes.

In designing and using network maps there is a particular terminology and a particular way of constructing them for route planning purposes. Network maps demonstrate how, for some purposes, a map may be a very simple representation used for a specific purpose.

Look at the map of the Singapore underground railway system (the Mass Rapid Transport or MRT) in Figure 14. The directions of lines and their twists and turns have mostly been removed. The purpose of the map is to help people negotiate the MRT system as quickly and easily as possible. The only important features are the stations and the connections between stations. Once again, information presented is highly selected so that particular data are stressed and others are completely ignored.

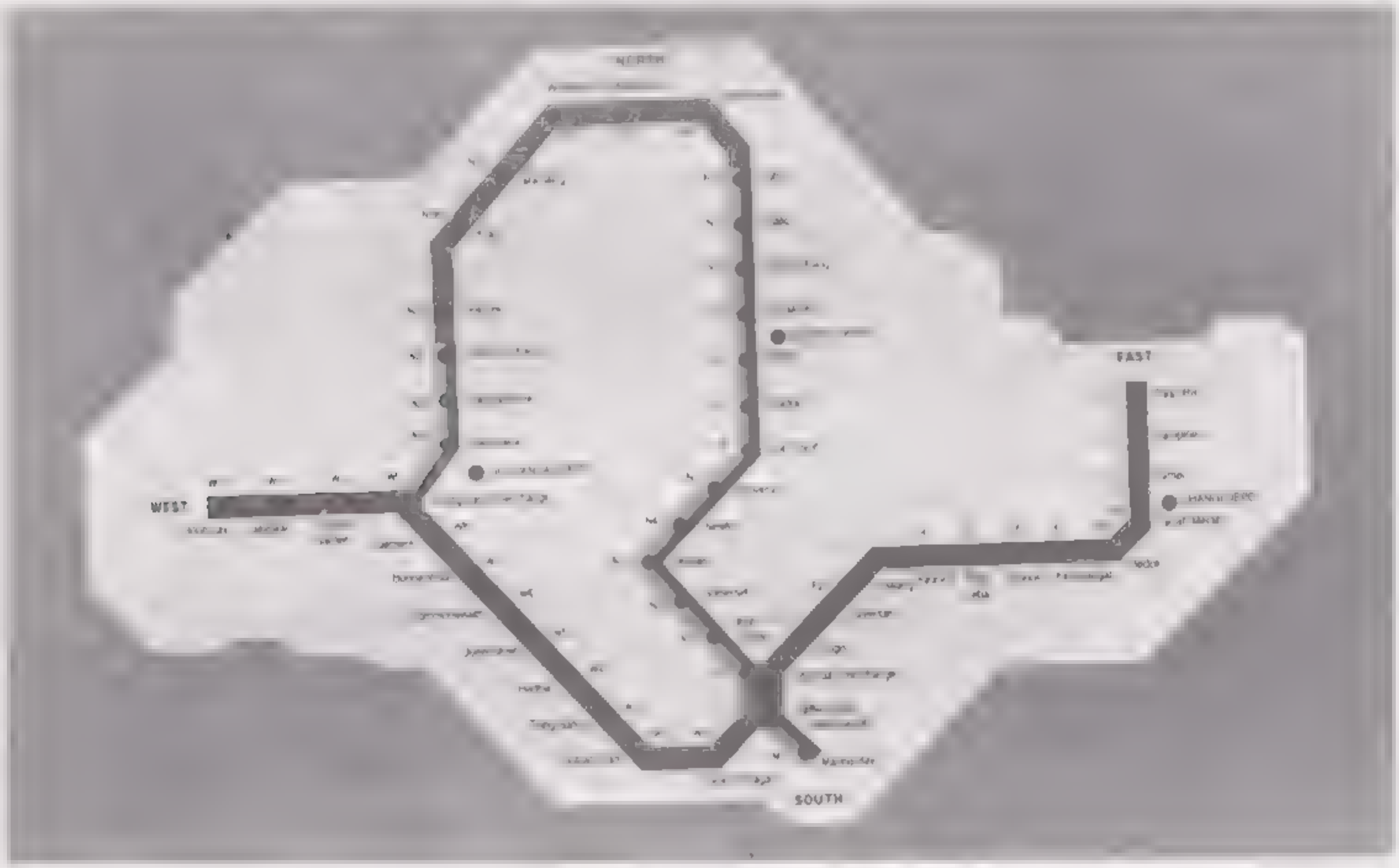


Figure 14 Singapore MRT map

The MRT map shows only the MRT network. Many rail networks produce such maps. For the MRT, there is no need to indicate the distance or time between stations as they are only a few minutes apart. Some road network maps do however show distances between junctions on the map, as do some railway network maps. Other railway network maps show the fastest times between stations.

InterCity Services

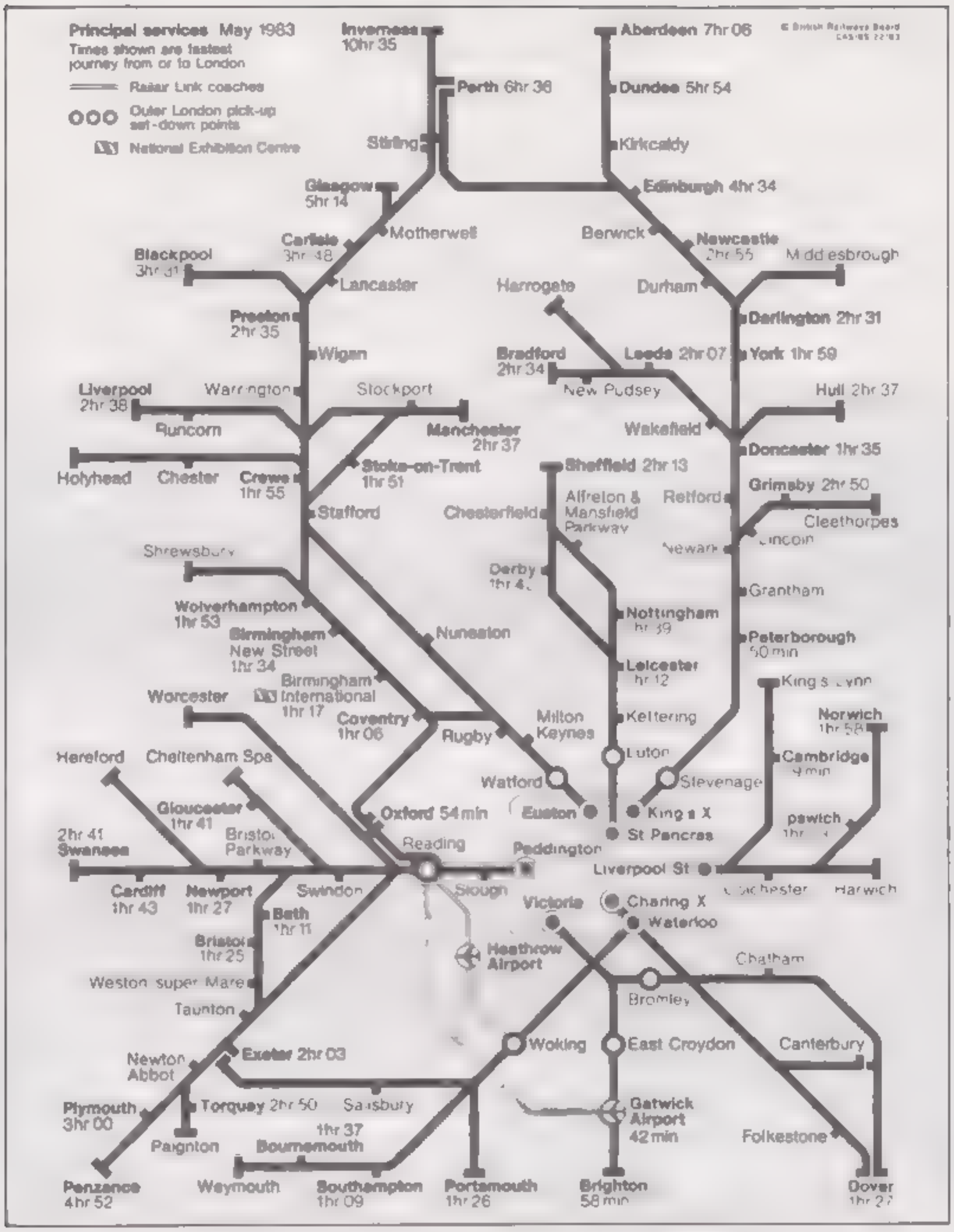


Figure 15 UK Intercity Services

Network diagrams can often be used to represent information contained on a map. The purpose is usually to aid route planning. If, for example, you are planning bus services to operate in a particular area, then a network diagram can be used to help the planning process. The UK Intercity Services diagram does pay some attention to showing relative directions, but again it clearly shows the data that have been selected to emphasize and create a particular image.

Defining a network map

Figure 16 illustrates how the network diagram represents what has been selected. Notice that detailed features such as bends in the road, bridges or other landmarks are omitted and that the towns are shown as points.

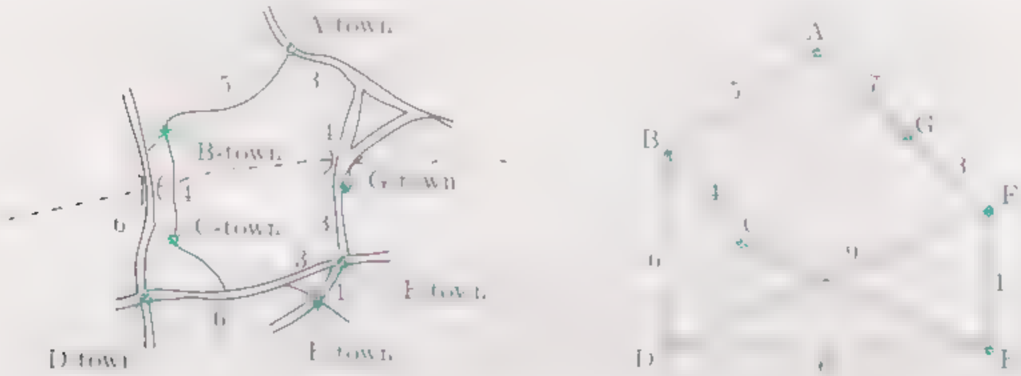


Figure 16 Simple map and network

Network diagrams consist of points, called *vertices*, and lines, called *arcs*, which connect them. If there are numbers on the arcs (for example, to represent the distance or time between vertices), then these are called *weights*. (You may remember the idea of using numerical weights from *Unit 3*.) The terminology is common to a number of applications. Figures 14 and 15 can now be seen as examples of transport networks.

Example 1 *Calculating using a network diagram*

Five towns, $A, B, \ldots, E$, are represented in the following network diagram. The 'weights' of the arcs are the distances, measured in kilometres, between towns. Notice that it is not drawn to scale. The result is, in fact, a simple mathematical model with only the connectivity information preserved.

A mathematical model simplifies the real world. There is more discussion on mathematical models in Subsection 4.2.

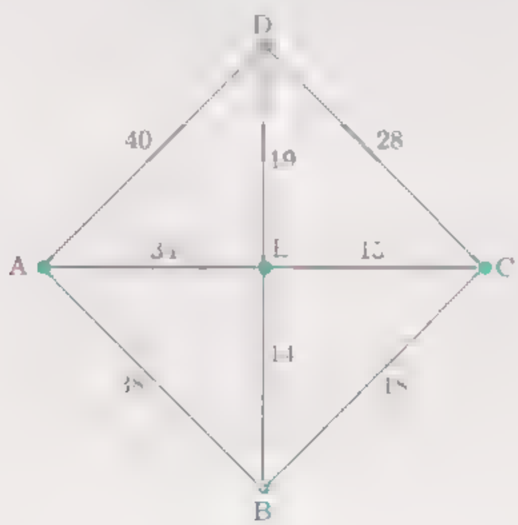


Figure 17 Simple network

What distance do you travel if you drive to B from D via C?

B to C is 18 km and C to D is 28 km, so the total distance via this route is 46 km.

Network maps help to chart progress and aid planning, but in producing them a great deal of information is thrown away, while the essential spatial detail, represented using vertices and arcs, is retained.

▶ Can you identify what is stressed and what is ignored?

Using mathematics to help design and analyse networks has enabled the quality of 'connectivity', that is, whether objects are or are not connected in some way, to be explored and used for many different purposes, from designing transport systems to constructing models of ecosystems.

Case study 5 High-tech maps

Among the sources of geographical data now available from which to compile representations [illegible] complex set known as 'Geographical Information Systems' or GIS. This is defined by a recent HMSO publication as 'A system for capturing, storing, checking, integrating, manipulating, analysing and displaying data which are spatially referenced to the Earth. This is normally considered to involve a spatially referenced computer database and appropriate applications of software.' It works by integrating all kinds of information into a single system where everything can be referenced in terms of geographical position.

The data, in digital form, can be displayed in a variety of ways: as maps, as photographs, in tables, by postcode. It is, at least potentially, a very powerful system. And it can be used for many purposes from traffic planning to land-use disputes to geological exploration. GIS can help to analyse some of the increasingly complicated issues that confront large organizations and governments. Monitoring the environmental consequences of a particular course of action may mean assessing literally millions of items of information, some of which are constantly changing. Yet it must be remembered that this too, like all other representations of the world, is a particular and partial view. Inevitably, not everything is included, even on such a huge database. What is included will reflect the interests of those who design, own, operate and support the system.

In the production of any representation choices and decisions have to be made. These decisions, however, will mean that particular points of view are presented in preference to the range of other possibilities: some information is stressed while other information is ignored.

Other representations

Representations stressing and ignoring information occur not only in the form of maps but in many other areas you may be familiar with. Indeed, you may have been creating a representation of this section as your study notes. Other representations include family trees, electric circuits, genetic histories, and the Periodic Table of the elements. Some representations may be special to you but all use symbols to represent often very complex and intricate data. And they are used to help identify patterns and make sense of vast amounts of information. Try to find some examples of representations that you use or come across.

Some groups who draw up their own family trees may wish to, or actually be in a position to suppress references to earlier marriages or relationships.

Activity 8 *Views about maps*

You have already by now accumulated a considerable amount of information relating to maps. To finish this section, study the reader article 'Images of the World' by Denis Wood. Do you agree with his views? Why does he comment that all maps are biased?

Now that you have studied Section 1, have your ideas and views about maps changed? Try to be as specific as you can in citing evidence. Add notes to your response to Activity 2.

Remember to complete Activity 5, relating to all the case studies, before finishing your work on the section.

To summarize, maps are created for many different purposes. Some examples of these purposes include: navigation (finding your way); communicating information, such as population density, climate; aiding work such as research into the spread of diseases or mathematical research; making political points, such as Australia should re-orientate itself away from the north towards the south.

Different conventions are used in maps: for instance, it is necessary to orient the user and nowadays north usually points to the top of the map. Conventions depend upon the purpose for which the map was created and also upon the person or people who created them.

The features which are included in a map depend upon its purpose, as do those which are excluded. For instance, the features needed in a tactile map of an area for blind or partially sighted people are different from those needed for sighted people: for example, steps or slopes are more important but visual signs are not important.

One of the simplest forms of maps is a network. Networks show points (vertices) and links between them (arcs). Sometimes the arcs have numbers (weights) associated with them. Network maps are useful to represent transport systems, such as movement by road or rail.

No representation of the world can be 'neutral'. Each representation necessarily has a particular perspective, stressing some things and ignoring others. You need always to be aware of this. Maps of the world frequently reflect the overall world-view at the time they were made.
There is a continuing debate about how the world should be represented. This has varied dramatically over time and continues today, even within the currently dominant scientific world-view.

At every stage in map production mathematics is an important and useful tool. Data collection, designing maps for a specific purpose, creating an accurate image, all use mathematical ideas and skills.

Outcomes

After studying this section, you should be able to:

- ◇ indicate the purpose of a map from the information it contains (Activities 2 and 6);
- ◇ discuss and describe the four properties of which any one can be preserved in a transformation from a 3D to a 2D image (Activities 3 and 4);
- ◇ identify features in a representation that are stressed and suggest some of those that have been ignored (Activities 5 to 8);
- ◇ read materials for a purpose and extract the appropriate information (Activities 3 and 8).

2 Mapping out a walk

Aims The main aim of this section is to bring out mathematical ideas such as the grid reference system, use of scales and contour line representations, that are embedded in maps. ◇

> The map is not an alien form that came from outer space but a synthesized system of supersigns we all grew up with ..., all of us *as a people*, and each of us ... *as individuals* ... .
>
> (Denis Wood (1993) *The Power of Maps*, Routledge, London, p. 144)

In this section and the next you are going to use an Ordnance Survey (OS) map extract to plan a walk in the Peak District National Park in the UK. An extract from the Ordnance Survey Outdoor Leisure Map 1—the Dark Peak area is included with this unit for you to use. If you visit this beautiful part of the country, you can try the walk yourself.

The Ordnance Survey

The primary responsibility of the Ordnance Survey is to survey and provide maps of Great Britain. The Survey was formally founded in 1791 with roots in the civilian and military map-making of the eighteenth century. In early Victorian England, the practical value of appropriate maps came to be appreciated not only for the management and transfer of land but also for civil engineering and urgent efforts to improve the sanitary conditions of the growing industrial towns. Scientific uses, including geological and archaeological mapping, also developed, so that by the mid-nineteenth century the Ordnance Survey was the established provider of maps of Great Britain for use by the public and the government. Today, the Ordnance Survey produces a wide range of maps for government, commercial, and public use.

(Based on Harley, J. B. (1975) *Ordnance Survey Maps—a descriptive manual*, Ordnance Survey, Southampton, p. 3)

As you work through this material, notice that it is not just about map reading —or about walking. It is also about noticing and identifying the mathematical features of maps and learning more about them so that you will be able to understand maps—and mathematics—better. Maps are a special type of pictorial representation, and the skills you will be developing are part of a more general mathematical language that uses graphical representations to communicate information.

You will all come to this unit with widely different experiences of maps. You may be an experienced walker and a confident user of OS maps—or you may be more familiar with street plans or road atlases. No prior

knowledge of any particular type of map is assumed here, although if some of the ideas are new to you it may take you longer to study the section than someone who has met the ideas before.

The next activity is designed to help you assess your experience and so adapt your study accordingly. You might like to try the activity as a collaborative exercise with others, especially if you or students you know are visually impaired or unfamiliar with OS maps.

Activity 9 *Familiar maps?*

Have you used an Ordnance Survey map before? Using the criteria 'very familiar', 'familiar' or 'not at all familiar', how would you rate your skill in the following activities:

- using grid references;
- using the map scale to work out distances;
- interpreting contours?

You may like to record your response to this activity on the sheet you used in Activity 2.

Now to get down to planning the walk.

Make sure you have the map [illegible] to hand as you work through this section.

The walk is in the area north of Castleton and follows the ridge which separates Castleton from the Vale of Edale. This area is towards the bottom of your map, and if you look carefully you might be able to pick out some of the landmarks on the walk: Mam Tor, Hollins Cross and Lose Hill. But this highlights the first problem. The map is very detailed and it may not be too easy to find a particular place.

2.1 Pointing by numbers

► How are particular locations specified accurately and unambiguously?

With the map in front of you it is no trouble simply to point to a name or a map feature and say 'That's where we're going' or 'This is where we are'. But how could you convey the same information to someone over the phone or in a letter? (You might like to try this some time with another student.)

Note that both sets of grid lines (horizontal and vertical) are parallels. By contrast, the lines of longitude which run from north to south in the global latitude and longitude system are not parallel. The lines of the National Grid do not correspond to lines of either latitude or longitude.

Maps produced by the Ordnance Survey use a reference system called the National Grid (not to be confused with the UK's electrical power distribution system). This grid, based on the metre as the unit of measurement, was established in its present form after World War II as a result of recommendations made in 1938. As Figure 18 shows, the National Grid covers Great Britain with a grid of horizontal and vertical lines 100 kilometres apart, subdividing the country into 100 kilometre squares.

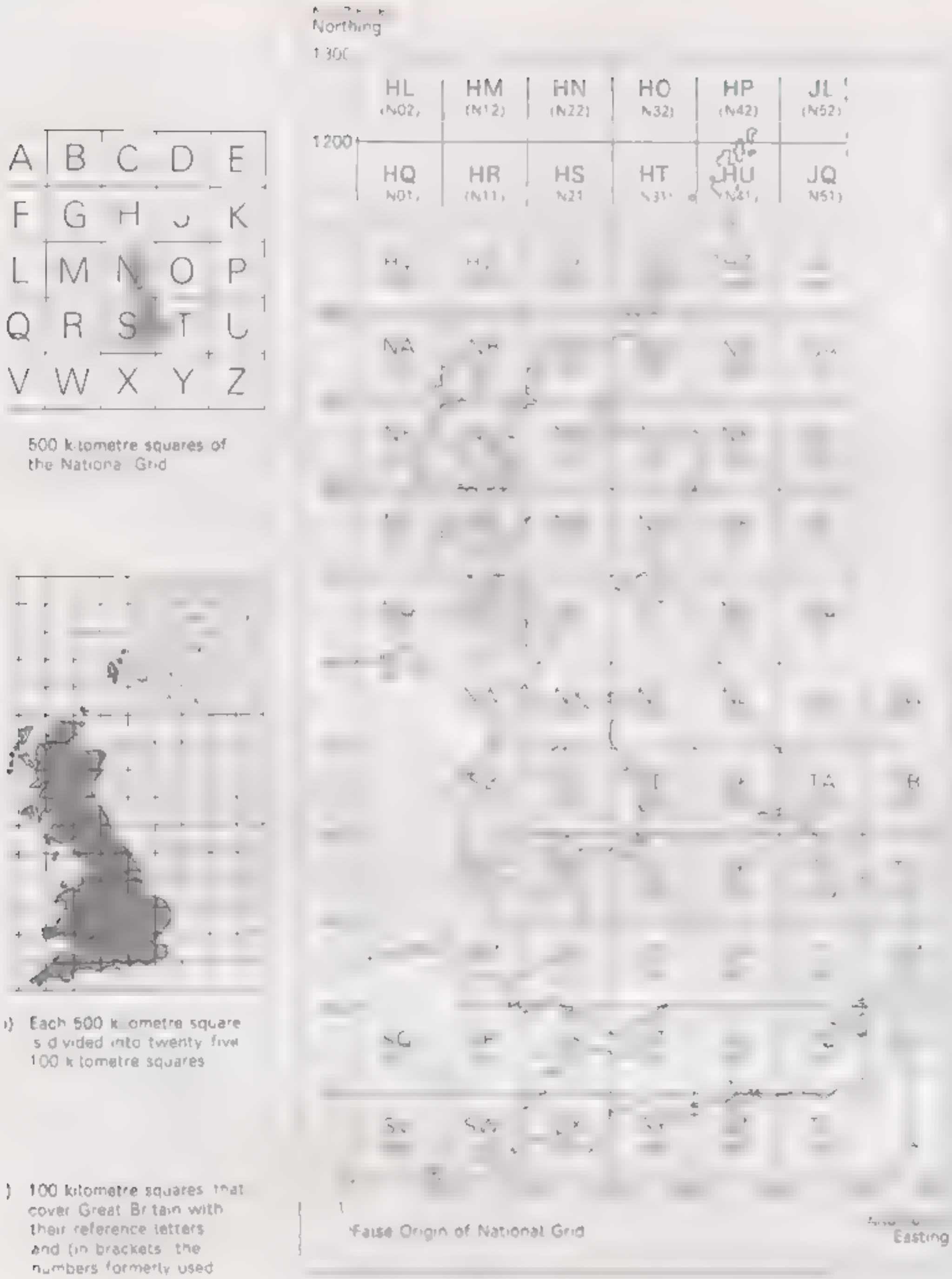


Figure 18 National Grid reference system of Great Britain

Because it is part of a larger 500-kilometre grid system, the origin is referred to by the Ordnance Survey as a 'false origin'.

Along the bottom and up the left-hand edge of Figure 18 you will see some numbers. The numbers along the bottom start at 0 and show distances to the east, these are called *eastings*. The numbers up the left-hand edge also start at 0 and show distances to the north; these are called *northings*. The starting point of the Grid, where both the easting and northing are zero, is called the *origin* and is shown at the bottom left-hand corner of Figure 18. Geographically, the origin of the National Grid lies about 100 kilometres to the west of the Scilly Isles.

The Grid forms a reference system, based on 100-kilometre squares, which can be used to locate any point in Great Britain. Where does your map fit in this system? Each 100-kilometre grid square is given a unique two letter reference. The Peak District occupies part of two grid squares *SK* and *SE*. You can specify these squares using eastings and northings.

The convention is to take the easting first. Look at Figure 18 again. Locate the vertical line that runs along the left-hand edge of the square *SK* and follow it down to the bottom of the diagram. You will find that it coincides with the number 400, indicating that the left side of the square is 400 kilometres east of the origin (which lies to the south-west of the square).

Now for the northing. Locate the bottom of the square *SK* and follow the line to the left. You will find that this line coincides with the number 300, indicating that the bottom of the square is 300 kilometres north of the origin.

The 400-kilometre easting crosses the 300-kilometre northing at the south-western (bottom left) corner of the National Grid square *SK*. In fact, this point defines the location of the entire square. The reference of the square could be written as 400 east and 300 north, but because eastings are always given first and northings always given second the reference is simply 400 followed by 300, written 400 300. These numbers are called the coordinates of the square *SK*. The space between the numbers is not strictly necessary but, for the moment, it helps to keep the two coordinates separate. Later, grid references will be given without the space.

Notice this space, used to separate the two values, is not the same use of a space as in the place-value representation of single numbers to show the thousands.

Activity 10 Grid coordinates

Look at Figure 18. Which grid squares are defined by the following coordinates?

(a) 200 700

(b) 600 300

What are the grid references for the following grid squares?

(c) *HY*

(d) *TL*

This system of reference based on distances from the south western origin defines the large 100-kilometre squares of the National Grid, but it is too cumbersome for planning a walk where you would probably want to know the location of places to within 100 metres or so. Moreover, individual grid references used on Ordnance Survey maps do not give the actual distances measured from the origin. Rather, grid references are distances measured from the south western corner of the relevant 100-kilometre square (and therefore are *relative* measures of distances).

Each 100-kilometre square is broken down into smaller squares. The upper part of Figure 19 shows how this works. The square *SK* is divided up into smaller squares by drawing grid lines running east–west and grid lines running north–south spaced at 10-kilometre intervals. This divides up the large 100-kilometre square into 100 squares, each one 10 kilometres × 10 kilometres.

Each 10-kilometre square is itself divided up into 100 smaller squares by drawing east–west and north–south grid lines spaced at 1-kilometre intervals, as shown in the lower part of Figure 19. The grid reference of each of these smaller squares is the distance in kilometres measured towards the east and towards the north from the south-western corner of the 100-kilometre square. Remember that a grid reference is the reference of a *square*, not a single point.

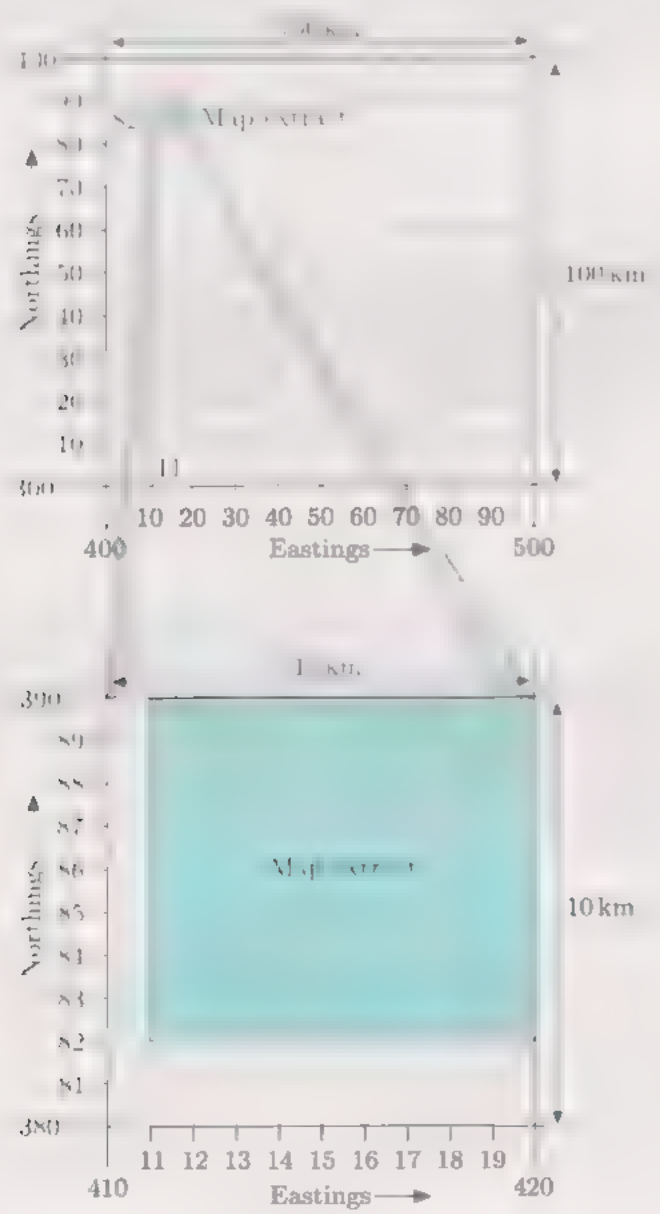


Figure 19

If you look at the south-west corner of your map, you will see the easting $^{4}11^{000m}$ on the southern (bottom) edge, and the northing $^{3}82^{000m}$ on the western (left) edge. These numbers are repeated on the top left and bottom right corners of your map. The first numbers from each of these, 4 and 3, are printed in smaller type and give the reference (in hundreds of kilometres) of the 100-kilometre square SK containing your map.

The numbers 11 and 82 printed in larger type give the distances in kilometres measured east and north from the south-western corner of the square SK, as in Figure 19. On your map, these numbers go from 11 to 20 along the top and bottom, and from 82 to 90 up the sides. They coincide with blue grid lines which run from side to side and from top to bottom. The grid lines are spaced at 1-kilometre intervals.

Now you should be able to appreciate the extent of the country covered by your map extract. Your map extends nine kilometres to the east and eight kilometres to the north from its south-western corner point. And this point is itself 11 kilometres to the east, and 82 kilometres to the north of the south-western corner of the square SK.

The smaller numbers following the easting 11 and the northing 82 complete the reference, in metres. Hence, the m at the end of the reference. Thus, the south-western corner of your map is 411000 *metres* east and 382000 *metres* north of the origin of the National Grid. In practice, this degree of accuracy is not used and grid references are usually only given to within 100 metres.

This way of giving a map reference is the one used by the Ordnance Survey. But it is not the only way. You have probably come across others such as the latitude and longitude system used in world atlases, or systems using combinations of letters and figures in street plans and road maps. Different conventions are used in different circumstances and it is important to know which one is being used in order to communicate information accurately and effectively.

Now you are in a position to use the grid reference system of eastings and northings to specify locations on your map. Ignore the numbers printed in smaller type in the references at the corners of the map, and consider only the eastings and northings relevant to the square SK. The eastings are the numbers printed against the grid lines along the top and the bottom edges. The northings are the numbers printed alongside the grid lines up the left and right-hand edges.

▶ Where do the grid lines with an easting of 13 and a northing of 84 cross?

The lines cross at the south-western corner of a 1-kilometre square. Locate this square. It contains Holms Cross, one of the viewpoints on your walk.

But you can be more precise than this. Along the edges of the map the distances between the grid lines can be subdivided further into ten divisions, where each division corresponds to 100 metres. Estimating by eye or by using a straight-edged object or ruler laid from the edge of the map, you should find that Holms Cross is just about six small divisions to the east of grid line 13, and just over five small divisions to the north of grid line 84, as shown in Figure 20. Holms Cross, therefore, has an easting of 136 and a northing of 845, so the six-figure grid reference is 136 845.

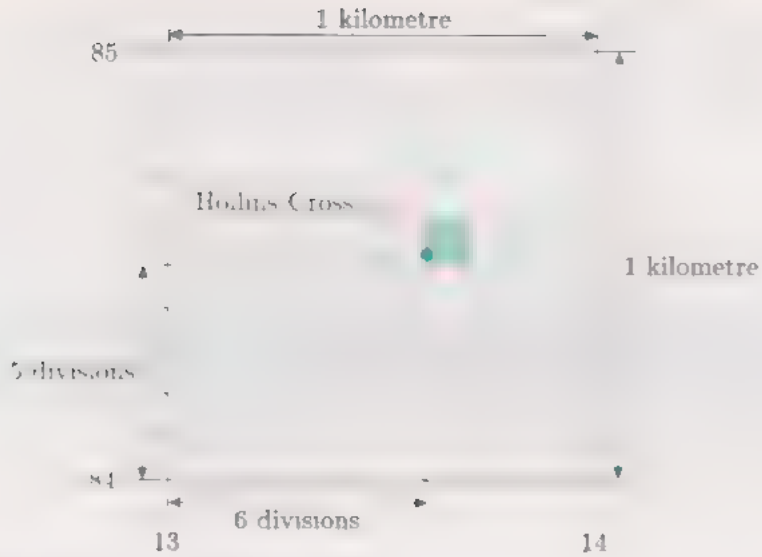


Figure 20 Grid reference of Hollins Cross

In an Ordnance Survey grid reference, the easting is always given first and the northing is always given second. By convention, the grid reference of a place is usually written as a single string of digits: 136845. Since each coordinate has the same number of digits—three in this case—there should be no confusion about which numbers belong to the easting and which to the northing.

When you are using grid references, bear in mind that a *six-figure reference refers to a 100-metre square* and not to a single point. All the points within the square share the same grid reference.

Locate Hollins Cross on your map and check the grid reference for yourself. For more practice in using grid references, try the next activity.

The full grid reference of Hollins Cross is SK 136845. The letters SK indicate the 100-kilometre grid square in which Hollins Cross lies. If the grid letters are not specified, then the reference is not unique—it could refer to a location in any 100-kilometre square in the country. However, leaving it out will not cause any problems within the area covered by your map.

Where a location lies between two 100-metre divisions, the correct grid reference is given by the lower, not the nearer, of the two. Thus, grid references always round *down* and never round up.

Activity 11 *Using a grid reference*

[illegible]
127836.

Now you should be in a better position to look again at the route of the walk. In the following description, the grid references are given in brackets. You will be focusing on a walk which starts near Mam Farm (133840). Find Mam Farm on your map.

From Mam Farm, the path to the south-west is shown as a green dotted line, changing to a black dotted line as it meets the route of an old road to the south, with Mam Tor lying to the west. The route follows the yellow road which curves to the west below the Tor, eventually meeting the A625 road (128831).

The route now follows the road to the west and joins a footpath (125831) near to a milepost, marked on the map as MP. The footpath goes north and then north-east to the top of Mam Tor (127836).

From Mam Tor, the walk follows the ridge to the north-east, passing Hollins Cross, Back Tor and on to Lose Hill. This ridge is popular with walkers and, when the weather is clear, offers spectacular views of the Vale of Edale to the north-west and of Castleton and beyond to the south and east.

From Lose Hill, the route follows the path down to Losehill Farm (158846), where the walk finishes.

Activity 12 *Finding grid references*

Use your map to find the grid references of Hollins Cross, Back Tor and Lose Hill.

Using grid references to specify locations on Ordnance Survey maps is a skill which you will have plenty of opportunity to practise as you work through this and later sections. From a mathematical point of view, a grid reference is an example of pointing by numbers—using a pair of numbers (called coordinates) to indicate a particular point on a two-dimensional surface such as a flat sheet of paper. It is a way of being precise. You will come across similar ideas about using coordinates when you learn about drawing and interpreting mathematical graphs in the next unit.

2.2 Symbols and scales

Your OS map represents information selected by the map-makers about the area around Castleton. But as a map-user, you also must be selective about the information you want. In planning the walk, for example, you are focusing on particular aspects of the map. Some features of the landscape, such as where the footpaths go, how steep they are and how far you must walk to get to a good viewpoint, are likely to be important. On the other hand, the routes of the major roads, railways and rivers are likely to be of less interest.

An OS map is not a picture like a photograph (see Figure 21) but rather is a structured collection of special symbols representing features of both the built and natural landscape. With practice you can learn to read these symbols and so gain access to some of the map's store of information.

A description of the symbols used on a map is given in a *legend* or *key*. The legend is at the bottom of your map sheet and gives a list of the symbols used to represent roads, paths, boundaries and rights of way, as well as natural features such as heights, rocks and vegetation. Because the map has been produced as part of an outdoor leisure series, this legend also contains symbols for camp sites and picnic sites, information centres and public conveniences.

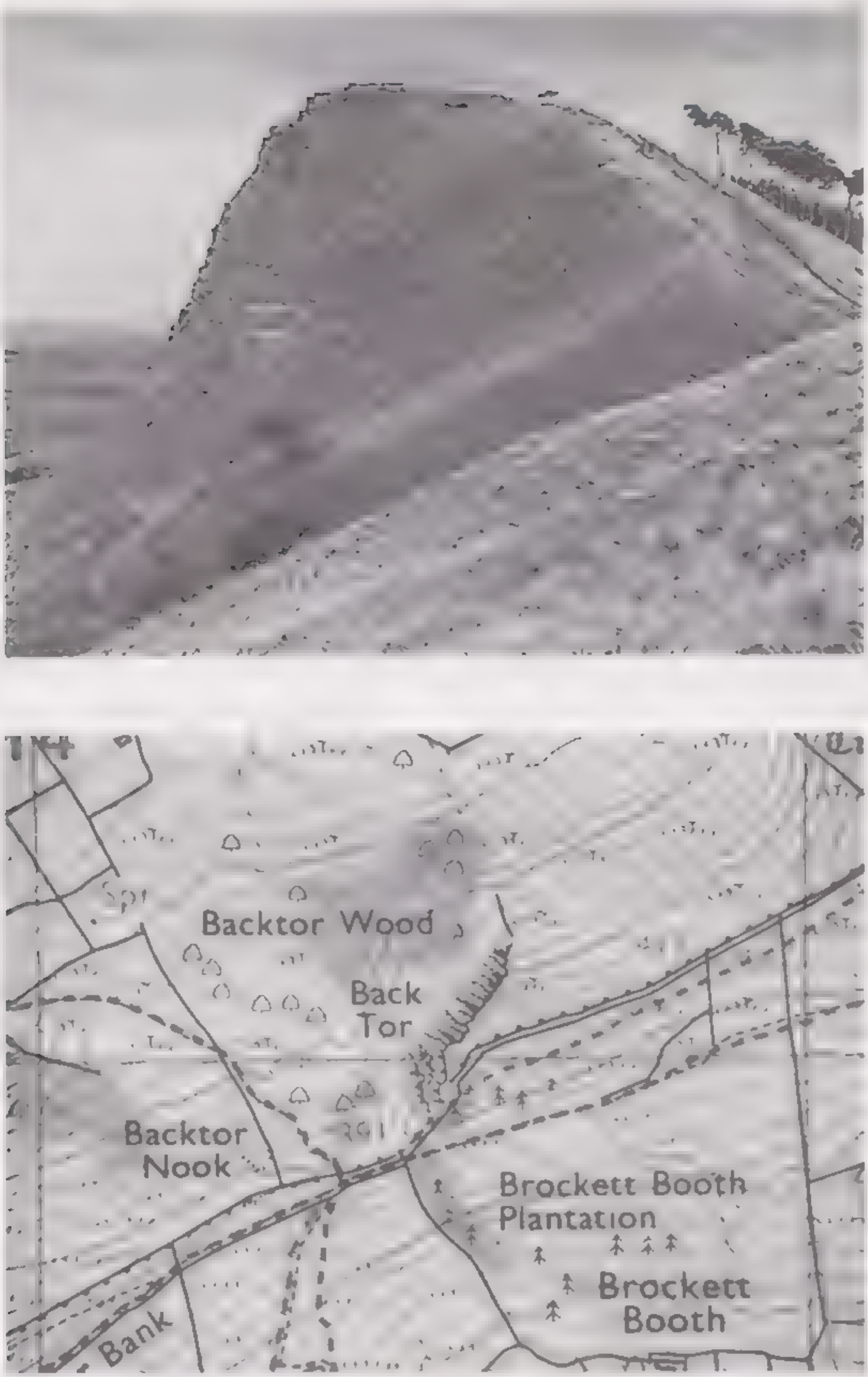


Figure 21 Two representations of the same feature: a photograph and a map of Back Tor

Activity 13 *Interpreting symbols*

Locate and identify, using the legend on your map if necessary, the features specified by the following grid references.

(a) 172834 (b) 181832 (c) 123832 (d) 140866 (e) 187851

On looking at a map, you are confronted by a range of different shapes, colours, special marks and text. Maps are complex symbols, not pictures of reality. But it is easy to get used to dealing with them as if they were reality itself, forgetting that the symbols on the map are simply marks on paper. 'This hill is steep' you might say, pointing at a pattern of closely-spaced brown lines, or 'Here's a road', pointing at a thick red or yellow line, or 'Here's a lake', pointing at a patch of blue ink.

'Here's a camp site' you might say, pointing at a strangely-shaped blue mark on your map. What you would mean is that if you were in that part of the country represented by the map, you would expect to find a place, probably with running water, showers and lavatories and possibly even a small shop, to pitch a tent for a few days or so. What you would not expect—contrary to what you already know about how the map-makers have chosen to represent water as patches of blue—is a smallish, strangely-shaped lake. Nor would you expect that the size of the symbol had any bearing on the size of the actual camp site, in spite of the fact that you probably know that maps are usually associated with some sort of scale. All this from a blue mark which, with a bit of imagination, seems to suggest a North American Indian tepee rather than the sort of tent most campers in the UK would actually use. Where has all this information come from?

A useful way of understanding what is going on in a map, and indeed in other complex representations, is to think carefully about what 'reading' a map means. One strategy is to split the notion of 'reading' into two separate processes. You can think of these as 'reading out' and 'reading in'. Reading out means identifying the information the representation gives you. So the blue tepee, or the thin brown lines, or the thicker red and yellow lines are all recognized explicitly as special symbols chosen by the map-makers. The borders of the map, the grid lines, the legend and the text in the legend are all part of this symbolic framework. Reading out means being aware of what the map is giving you, and the context—that is, the conventions and traditions associated with this type of Ordnance Survey map—in which the map has been produced.

Reading in, on the other hand, involves you bringing your own knowledge and experience to the map, and interpreting the map so that it has meaning for you. You learn to 'see' the overall map as a representation of a landscape and the symbols as telling you about places or features of that landscape—camp sites, picnic sites, churches, roads, wooded valleys and steep hills.

The text in the legend may trigger off interpretations. The blue tepee shape has the words 'Camp site' next to it. But this is not an explanation. You have to know that in this context the camp site is for tourists, and not, for example, one reserved for travelling people. And you have to know about how tourist camp sites are usually run and what sort of facilities they offer—all this before the words carry very much meaning for you. Maps are cultural objects; they make sense only if you can give the symbols meaning. To someone who does not know about camp sites—or perhaps about this particular *type* of camp site—the legend is pretty uninformative.

Maps and map symbols can become so familiar that it can sometimes be hard to remember that they are 'only' symbols and not the things themselves. Despite the apparent wealth of information on a map of an area, there is still an unlimited amount more that could be said. For example, the map does not indicate what you can or cannot see at ground level. A church clearly marked on the map may be obscured by trees. A post office may have closed down. Footpaths may have been rerouted or no longer exist. Rural as well as urban areas, woods as well as industrial sites, may have changed. The map is not the country.

The map legend tells you that the map was compiled from larger scale surveys done between 1953 and 1979, and then revised several times more recently. Although the information included on the map is intended to be as accurate as possible, it reflects the revision and updating policies of the Ordnance Survey as much as reality itself. The map is not a snapshot of the area taken at a particular moment, but a symbol which has evolved through time. It is as much a record of the influence of interests (governmental, commercial, public) on those involved in producing the map as it is of the area it claims to represent.

Today, maps and charts of all types can be updated electronically using Geographical Information Systems (GIS)—see page 32.

▶ Do you agree? Stop to think about this for a moment.

Activity 14 What is ignored?

Write down some of the things that occur to you that the map does not show. You may find it helpful to refer back to Section 1.

Scale

One of the most important uses of many, but by no means all, maps is to represent distance. Your OS map uses a scale to relate distances measured on the ground to distances measured on the map.

Look at the map legend. At the top is printed:

Scale 1 : 25 000

This statement gives the scale of the map. It is written as a ratio and should be read:

1 unit of distance measured on the map represents 25 000 of the same units measured on the ground.

Notice that it does not say whether the measurements should be in centimetres, inches, metres, yards, kilometres, miles, or some other unit. This is because it does not matter which units you use, because it is a *ratio*, as long as you use the same one for both measurements.

If you are using centimetres, then 1 centimetre on the map represents 25 000 centimetres (250 metres) on the ground. On the other hand, if you are using inches, then 1 inch on the map represents 25 000 inches (about 694 yards or 635 metres) on the ground. Note that 'measured on the ground' means that the distance is measured along flat ground: the horizontal distance between any two points.

To calculate the distance on the ground, you multiply the distance on the map by the map scale. The relationship can be expressed as a word formula:

$$\text{distance on the ground} = \text{map scale} \times \text{distance on the map}$$

where both distances are measured in the same units. For a 1 : 25 000 map such as yours, the formula to work out ground distances from map distances is:

$$\text{distance on the ground} = 25\,000 \times \text{distance on the map}.$$

Or, by rearranging the formula, map distances can be worked out from ground distances:

$$\text{distance on the map} = \frac{\text{distance on the ground}}{25\,000}.$$

Remember that these relationships are valid only if the map and the ground distances are expressed in the same units.

Using the map scale on its own, however, is often not the easiest way to think about the relationship between map distances and ground distances. In everyday language, map scales are usually said to be 'so many centimetres to a kilometre', or 'so many inches to a mile'. This is a useful way of thinking because it allows you to estimate by eye the actual distances between places when you look at a map. With a bit of practice, it becomes relatively easy to look at a familiar map and judge distances fairly accurately.

To help you to estimate distances easily your map shows two linear (straight line) scales marked out directly in metres and kilometres in one case, and yards and miles in the other. Both show what a scale of 1 : 25 000 means in everyday terms. If you use a ruler to measure the scales, you will find that there are 4 centimetres to a kilometre, or just over 2½ inches to a mile.

Activity 15 *Using a map scale*

Assume a map scale of 1 : 25 000.

(a) A footpath between two villages measures 12 centimetres long on the map. How far would it be to walk between the villages?

(b) A road between two places is 7.5 kilometres long. What length would you find if you measured it on the map?

Maps are often referred to as 'small scale' or 'large scale'. A small-scale map is one on which relatively long distances on the ground are represented by relatively short distances on the map. So, for example, a map drawn to a scale of 1 : 25 000 is at a smaller scale than one drawn to a scale of 1 : 2500. A distance of 1 kilometre on the ground would be represented by a distance of 4 centimetres on a relatively small scale 1 : 25 000 map, but represented by 40 centimetres on a larger scale 1 : 2500 map.

Estimating distances from a map is particularly useful in planning a walk. You may want to know the distances from point to point along the route so that you can judge how long each section will take. And almost certainly you will want to know how far your proposed walk will be from beginning to end to make sure it is within your capabilities. You can do this in stages.

Activity 16 *Calculating distances*

Look back to Subsection 2.1 where the walk is described. Find the start of the walk at Main Farm (133840) on your OS map. How far is it from there to the point where the route reaches the A625 road (128831)? The route is not a straight line, so you will have to estimate the map distance, perhaps using a piece of paper or string, or even an opisometer (a small map-measuring device) if you have one. Then use the map scale to convert the map distance to ground distance in kilometres. Try this now.

Note particularly how you convert your map distance to your predicted actual distance.

The walk now follows the A625 road west until it meets a footpath at (125831). On the map, this distance is about 1.2 centimetres, corresponding to a distance of $1.2/4 = 0.3$ kilometres along the road.

You need to repeat this exercise for each stage of the walk. One way to organize the information about distances is by drawing up a table. Table 1 contains a partially completed list showing each stage and the corresponding map and ground distances.

Table 1 Table of distances for the walk

From Grid reference	To Grid reference	Measured map distance in centimetres	Distance calculated on ground in kilometres
Mam Farm (133840)	A625 road (128831)	4.5	1.13
A625 road (128831)	Footpath (125831)	1.2	0.3
Footpath (125831)	Mam Tor (127836)	2.4	0.6
Mam Tor (127836)	Hollins Cross (136845)	5.3	1.33
Hollins Cross (136845)	Back Tor (145849)	4.3	
Back Tor (145849)	Lose Hill (153853)	3.5	
Lose Hill (153853)	Losehill Farm (158846)		
Total distance			

Activity 17 *Predicting distances*

Complete Table 1 by measuring the distance on the map for each leg of the walk and converting to kilometres. You can use the map scale or the kilometre scale shown at the bottom of your map.

How far is it from Mam Farm to Losehill Farm?

2.3 *No sign of the hills*

> But everyone agrees that *hills* are the hardest thing to get right on a map.
>
> (Denis Wood 1993 *The Power of Maps*, Routledge, London, p 144)

Visitors to the Peak District are attracted by the magnificent tors (rocky outcrops), peaks and high moors in short, by the hills. Look at your map of the Peak District now.

▶ Where are the hills?

Of course, there are none. All you have in front of you is a flat sheet of paper covered in coloured ink. What the question should have been, perhaps, is: 'How are the hills represented on the map?' You may remember from Section 1 that one of the problems faced by early map-makers was how to represent the relief of the terrain. The hills are not represented, at least not as such. There are no special symbols which stand for hills. What you are given is information about height, but given in such a way as to make possible other deductions about the nature of the landscape.

Stop for a moment and think about how you would go about showing something of the three-dimensional landscape on a two-dimensional sheet of paper. This is a problem that has been tackled by different cultures in a

variety of ways over the years. Figure 22 shows a chronology of some of the ways that hills have been depicted graphically from the signs used by the Mixtec and Nahuatl peoples of pre-Hispanic Mexico some 6000 to 7000 years ago, to the more familiar representations in use today.

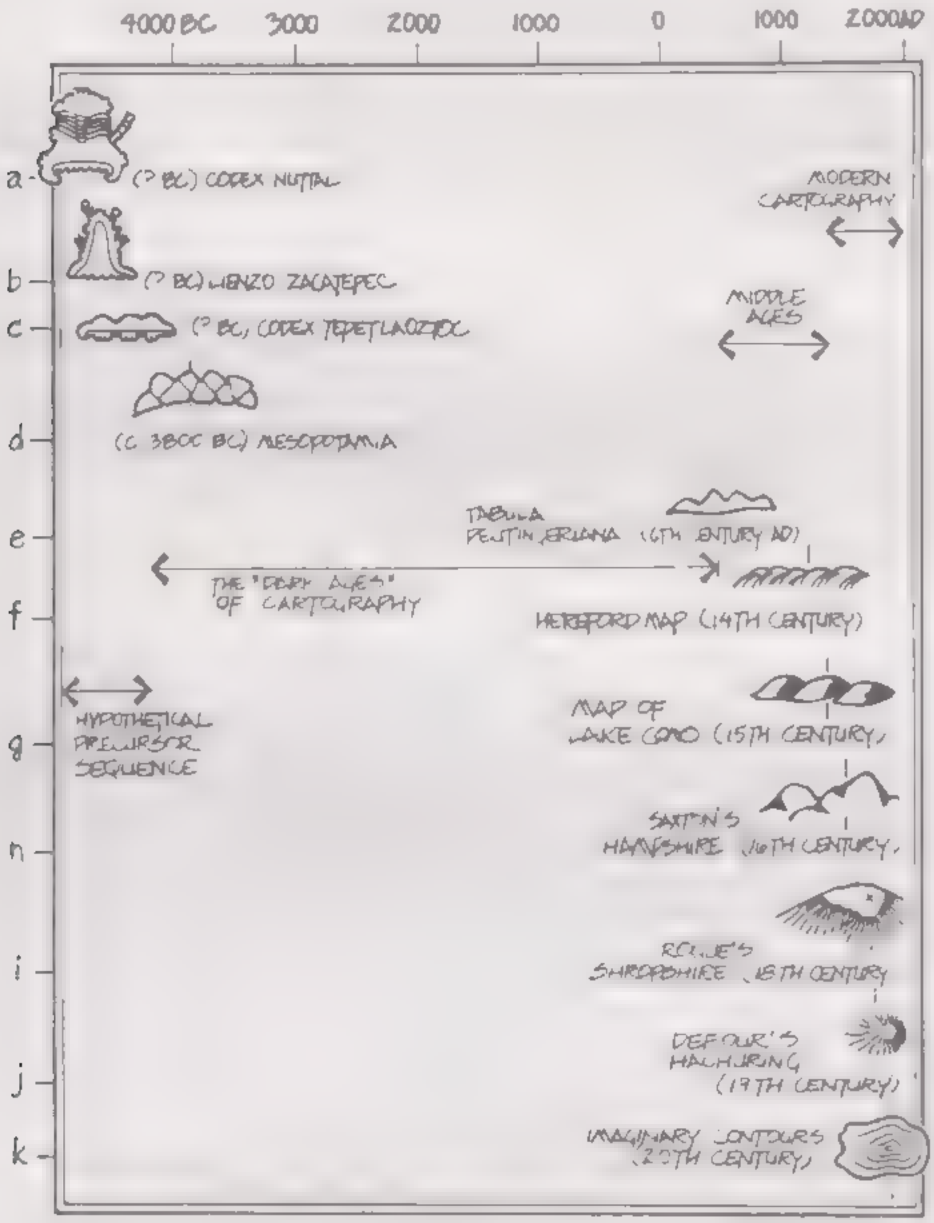


Figure 22 A chronology of hillsigns (from D. Wood, *The Power of Maps*, p. 153)

Figure 22 seems to suggest that over the ages there was a move from pictures of hills (although these were clearly very stylized and not necessarily pictures of any particular hill) to more abstract representations which do not resemble hills at all. This process of abstraction is one which you will also come across in mathematics, where diagrams, graphs, symbols and formulas usually do not resemble or look like the things they are representing in any way.

Central to the notion of abstraction is that of capturing the features you want to stress, while ignoring those aspects you consider less important. In the historical development of cartography, a major step in the representation of relief—the three-dimensional shape of the landscape—was to move away completely from specific symbols for hills and valleys. Instead, what was to be shown on the map was a more abstract concept—that of height.

Height has to be measured from somewhere. A statement of height on its own is meaningless unless you know where the height is measured from. In the UK, the Ordnance Survey measures height relative to a fixed level, called the Ordnance Datum. On an OS, map all heights are quoted relative to the Ordnance Datum.

The Ordnance Datum (OD) is the mean level of the sea at Newlyn in Cornwall. The mean was calculated from hourly measurements of the sea-level taken between 1 May 1915 and 30 April 1921. Recall from *Unit 2* that 'mean' means the arithmetic mean, an average value of the data.

Height information expressed simply as numbers is not enough to give a visual indication of the lie of the land. Figure 23, for example, shows a tourist map of the Black Mountains region of Brittany in France. There is plenty of height information if you care to look for it, but it is presented as spot heights—the heights at particular points. It is difficult to gain any visual impression about the shape of the landscape from this representation.

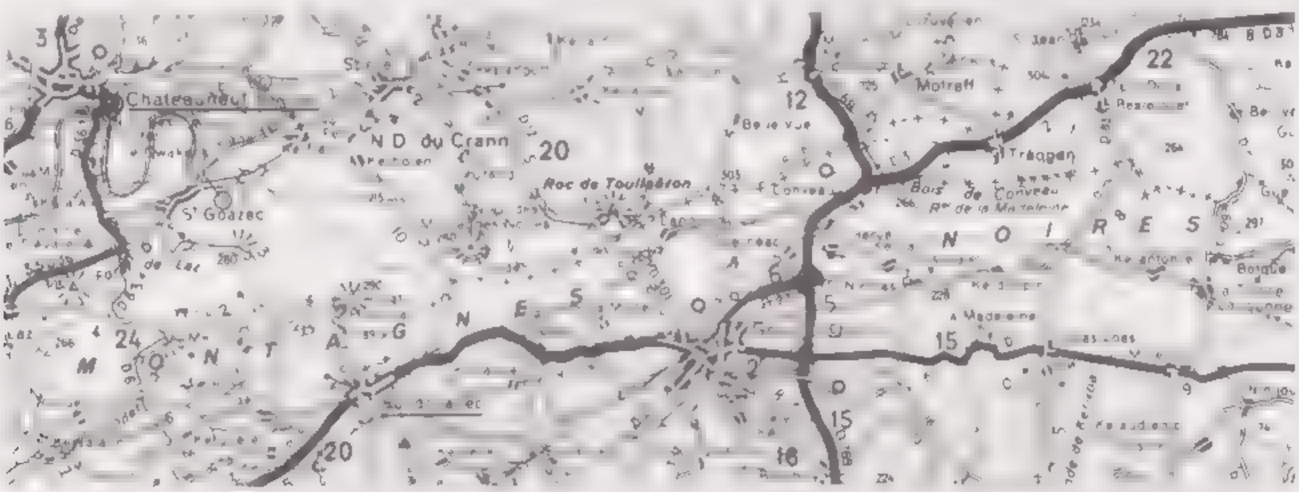


Figure 23 The Black Mountains region of central Brittany

You will also find spot heights on your OS map, but they do not carry the information about the shape of the countryside. That task is accomplished by the brown lines called *contour lines*. Each contour line joins points which (on the ground) have the same height above mean sea-level. If you were to walk along the path of a contour line on the ground you would remain at exactly the same height above the Ordnance Datum. (Of course, not all maps show contours. Contours serve a purpose. You might like to spend a few moments thinking about whose purpose they serve. Why in fact does the Ordnance Survey produce contour maps?) Contours are clearly useful on a tourist leisure map for walkers; they are less important on street plans or road atlases.

The contour lines on your map are drawn at 10 metre intervals. On the east side of Mam Tor, for example, there are lines representing heights of 350 metres, 360 metres, 370 metres, and so on. Where the lines are well spaced, you should be able to see the height marked somewhere on the line. Where the lines are close together, such as near the top of the Tor, only a few labelling heights are printed. By counting in 10-metre steps from the nearest known contour, you can work out the height represented by an unmarked line. The contour lines at 50-metre intervals are slightly thicker. You can, perhaps, pick out the lines at 250, 300, 350, 400 and 450 metres on the east side of Mam Tor.

The way the numbers giving the heights are printed on the contour lines indicates the direction of the slope from the point of view of the map reader. Look at the map in its normal orientation with north at the top. The contour heights are printed so that the tops of the numbers point towards the top of a slope. Look at the way the numbers are printed in the 1-kilometre square 1487. They tell you that the ground *rises* as you move north-west to Upper Moor. In contrast, look at square 1188. From Seal Stones, the ground slopes *down* to the north-east, indicated by the contour heights appearing upside down as you look at the map.

The steepness of a slope is indicated by the spacing of the contour lines. If the ground rises or falls steeply so that the height changes rapidly over a small horizontal distance, the contours will be close together. Where the ground is flatter, so that the height changes more slowly, the contour lines will be more widely spaced.

Here are some examples of contour line patterns. The left-hand side of Figure 24 shows a pattern of concentric rings. The numbers indicate the rising and falling slopes, with the highest point at the centre. The pattern represents a bird's eye view of a hill, with each contour line indicating the points around the hill that are at the same height.

On your map you can see a similar pattern of lines around Lose Hill (153853

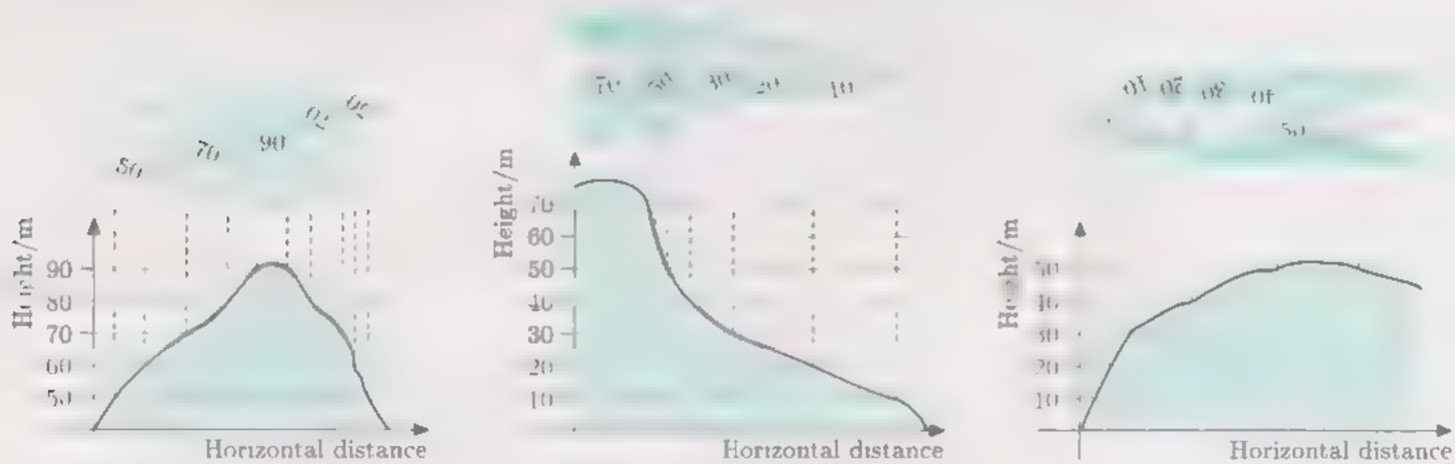


Figure 24 Different patterns of contour lines for different features

In the middle and right-hand side of Figure 24, you can see how the spacing of the contours reflects different types of slope. Contours bunching up indicate a slope that is getting steeper. In the middle one, the bunching occurs near the top of a slope showing that the steepness increases with height. In the right-hand one, the bunching occurs near the bottom, indicating that the lower slopes are the steepest.

Activity 18 Weather maps

The word 'isobar' comes from the Greek prefix *iso* meaning 'equal' and the word *baros* meaning 'weight'. A *bar* is a unit of pressure. In meteorology, atmospheric pressure is quoted in millibars. One bar is 1000 millibars.

OS maps are not the only place you will find the contour idea being used. On weather maps, lines called *isobars* are drawn to connect points at which the atmospheric pressure at sea-level is the same. The pattern of isobars indicates how the pressure varies. Numbers on the isobars give the pressure in millibars.

Look at the weather map in Figure 25. At what intervals are the isobars drawn? How would you describe the variation in atmospheric pressure at sea-level along the coloured line marked on the map?

L means 'low pressure' and H means 'high pressure'.

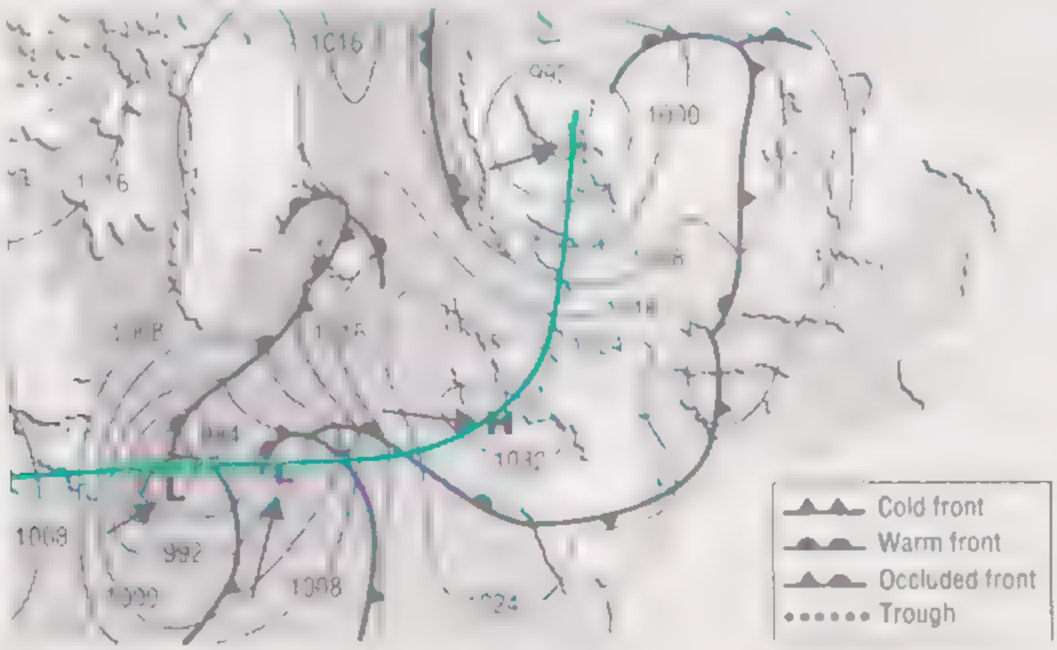


Figure 25 Weather map showing isobar 'contours'

Like an OS map, the weather map uses a contour system which can be read to give the 'shape' of the peaks and troughs in the air pressure. The pattern and spacing of the contour lines adds extra information to the map. With a little practice, you can learn to read contour-line patterns and visualize the three-dimensional shape of the landscape.

So far, you have looked in a fairly general way about slopes and changes in height. Now focus on a more mathematical description. Look again at the

contour lines to the east of Mam Tor. If you measure 2 centimetres on the map to the right (that is, due east) of the summit of Mam Tor, you will be close to the 310-metre contour line. 2 centimetres on the map represents a ground distance of $2 \times 25\,000 = 50\,000$ centimetres, or 500 metres. In that distance, the map shows that the height of the ground has fallen from 517 metres to 310 metres: a drop of 207 metres. In other words, for each metre towards the east away from the summit the ground level drops on average by 207/500, or about 0.4 metres. This is a steep slope.

Down in the Vale of Edale, however, the ground is considerably flatter. This is reflected on the map by much more widely spaced contour lines. Near to the symbol for Edale church (grid reference 123857) is the 250-metre contour. Measuring on the map 2 centimetres to the left, corresponding to a ground distance of 500 metres to the west, brings you close to the 290-metre contour. In 500 metres, therefore, the ground has risen about 40 metres. This corresponds roughly to a rise of 8 centimetres (or 0.08 metres) for every metre to the west from the church: a fairly gentle slope.

Imagine now that you are standing on the top of Mam Tor (127836). Locate this point on your map. The ground slopes away steeply on all sides except to the north-east, the direction of the path along the ridge to Hollins Cross. The contour lines crossing the line of the path are less closely spaced than they are on the other sides of the Tor. The height of the ridge path drops as you get closer to Hollins Cross. You might just be able to make out the 390-metre contour near the Cross. This point is nearly 130 metres lower than the summit of Mam Tor, but since the height falls over a distance of about 1.3 kilometres, the slope of the path is nothing like as steep as the north and south sides of the ridge. This part of the walk should cause no major problems.

On towards Back Tor, the path skirts the high point at 426 metres, marked as a spot height on your map, and follows the contours, reaching a point where several paths meet. At Back Tor itself, the contours are bunched together indicating a sharp increase in the slope: in a short distance, the path rises about 30 metres from 391 metres to just over 420 metres.

▶ If you could cut vertically downwards along the line of the path, what would the profile of the cut edge look like?

Figure 26 shows the profile of the path, shown from the Brocket Booth side and looking north-west towards the path. This section of the path will be a short but steep climb up to the top of Back Tor.

After Back Tor, the last stop on the ridge is Lose Hill. Notice the distinctive pattern of the contours indicating the shape of Lose Hill.

▶ Which side of the hill has the steepest slope, and which has the gentlest slope?

The north-west side seems to be the steepest and the south side is the least steep as the contours are more widely spaced.

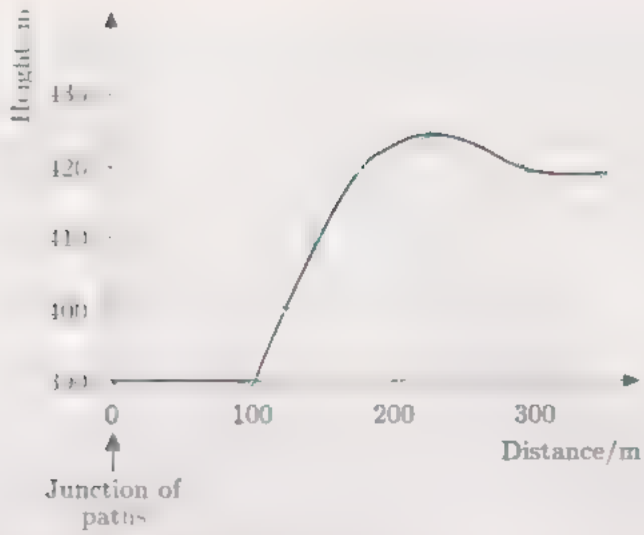


Figure 26 Profile of the path up Back Tor

Later you will learn more about working out gradients and slopes. For the moment, however, try the next activity to give you a feel for working with contour lines.

Activity 19 *Sketching profiles*

Follow the path from Back Tor to Lose Hill on your map, noting where the contour lines cross the path, and what heights they represent. Make a sketch of the profile of the path.

Indicate how you expect the steepness of the path to vary as you approach Lose Hill, and remember to include a legend if you use your own symbols.

You have now almost completed this section, so this is a good point to draw some ideas together.

In Activity 17, you completed a table of the distances of the different stages of the walk. The completed table is shown below. You can see that the total distance from Mam Farm to Losehill Farm is nearly 6.2 kilometres.

Table 2 Distances along the walk

From Grid reference	To Grid reference	Measured map distance in centimetres	Distance calculated on ground in kilometres
Mam Farm (133840)	A625 road (128831)	4.5	1.13
A625 road (128831)	Footpath (125831)	1.2	0.3
Footpath (125831)	Mam Tor (127836)	2.4	0.6
Mam Tor (127836)	Hollins Cross (136845)	5.3	1.33
Hollins Cross (136845)	Back Tor (145849)	4.3	1.08
Back Tor (145849)	Lose Hill (153853)	3.5	0.88
Lose Hill (153853)	Losehill Farm (158846)	3.4	0.85
Total distance		24.6	6.17

Using a table to record distances is one approach, but can you think of another way? Before reading on, look back at Section 1 of this unit.

▶ What type of map could you use to carry place and distance information?

An alternative approach is to sketch a network map of the walk and write the distances on the links joining the points. Look at the network map in Figure 27. You can see that it stresses the main places along the walk and the distances between them. All other features of the route are ignored. The map represents the path as a series of straight lines, but there is no attempt at a scale or at providing accurate geographical relationship between the places.

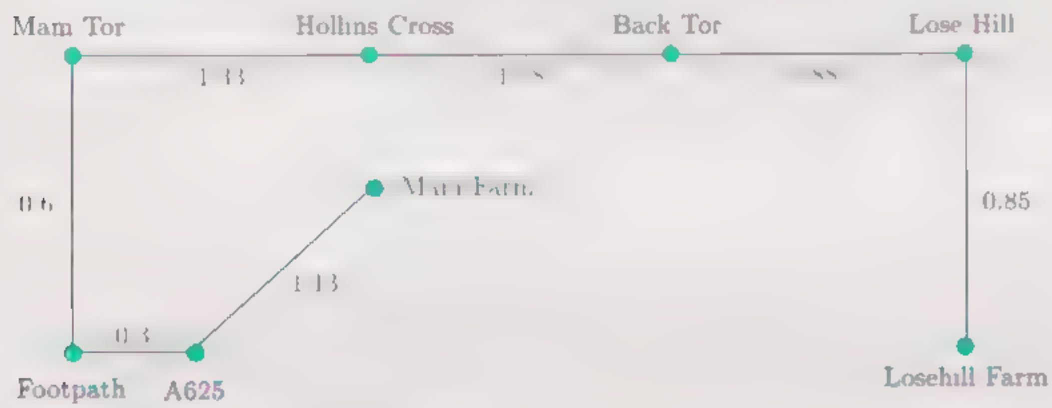


Figure 27 Network map of the walk

The table and the network map are two different representations of the walk.

▶ Can you suggest any relative advantages or disadvantages of each representation?

An advantage of the network map is that it gives an immediate visual impression of the walk. It is straightforward yet it contains the main items of information: place names and distances. On the other hand, there is no way to check whether the distances on the network map are correct. You may have made a mistake in converting from map distance to ground distance. The table allows you to check both these figures if there is any doubt, but the network map contains insufficient information to carry out such a check. Of course, if you made a mistake in measuring the map distance in the first place, this error will be built into the table, too. But the point is that the simplicity of the network map is traded off against the more complex but checkable table. In a mathematical representation, leaving out information may lead to a clearer presentation of important results, but may make the job of checking and correcting errors more difficult.

Recall from *Unit 5* the features of the boxplot as a representation of a batch of data.

So you now know how far you will be walking. In the next section, you will use the map to find out about directions and bearings, and to estimate how long the walk will take.

Activity 20 *Completing the Learning File activity*

Before leaving this section, there are two activities to complete.

First, go back through this section to add to the activity sheet you started in Activity 5. Think about the main techniques you have been using, such as using coordinates to locate precisely; using ratios and scales and their conversion; interpreting height data on OS maps. Summarize the main points to help you remember and consolidate your learning.

Second, look back to Activity 2 that you started in Section 1. Can you add anything to your notes?

A map is a symbolic representation of selected information which you can learn to read. The main aim of this section was to bring out some of the mathematical ideas that are embedded in your Ordnance Survey map. As such, you have been involved in exploring how mathematics can help in interpreting representations. These ideas include a two-dimensional easting and northing coordinate system for specifying position, scales to convert from one set of measurements to another, symbols to convey meaning, and contour lines to provide extra information about heights and slopes.

Outcomes

After studying this section, you should be able to:

- appreciate that a map is a symbolic representation of reality, stressing some aspects, ignoring others (Activity 14);
- use the following terms accurately: 'grid system', 'map scale', 'legend', 'key', 'contour line', and explain them to someone else unfamiliar with them (Activities 10 to 13,15 and 19);
- use grid references to specify a location on an OS map (Activities 10 to 13);
- convert measurements of distance on the ground made at one scale to their corresponding values at another (Activities 15, 16 and 17);
- explain how contour line patterns are used to represent three-dimensional shapes on a two-dimensional map, and give some examples of contour patterns (Activities 18 and 19).

3 Getting your bearings

Aims The main aim of this section is to introduce some processes involved in mathematical modelling in the context of maps. It also aims to help you develop an understanding of angles and direction bearings, and to appreciate how bearings can be used to specify a location. ◇

This section is in three parts. First, it uses a video band to show you the route of the walk you mapped out in Section 2 to help you appreciate links between the features of the landscape and the symbols, conventions and patterns used on the Ordnance Survey map. The video band 'Getting your bearings' is intended to help you visualize the three-dimensional landscape from the two-dimensional map. It is an important part of the study material for this section and you should watch it as part of Subsection 3.1.

In Subsection 3.2, you will need your OS map extract and a protractor to measure angles. If you are unsure about using a protractor, you should listen to band 2 on Audiotape 2, when you reach the appropriate subsection. The tape frames are in the appendix on page 105.

Subsection 3.3 looks at mathematical relationships, and writing down word formulas to express them. You have already started to do this with map scales. The discussion focuses on a rule of thumb, called Naismith's rule, used by walkers and climbers to estimate how long a particular walk will take. You are encouraged to think about the assumptions Naismith's rule uses and the accuracy of its estimates.

3.1 Hill views

This subsection includes watching the video band 'Getting your bearings'. The video lasts about twenty-five minutes and gives different perspectives on the walk you started planning in Section 2.

One perspective is to see the route of the walk through the eyes of three walkers. You will be able to compare the shape and features of the actual countryside with the image you may have in your mind's eye after working with the map. Not all the features the walkers come across are marked on the map, underlining the message that a map is a selective representation of an area and not simply a picture of what is on the ground.

A different perspective is provided by the map makers of the Ordnance Survey. The video also shows how computer-generated graphics are used at the Ordnance Survey to relate the patterns of contour lines on maps to the heights, slopes and overall shape of the actual countryside. You will be able to see the path of the walk marked out on a three-dimensional view of the ridge from Mam Tor to Lose Hill, and be able to compare the geographical features of places such as Hollins Cross and Back Tor with the patterns of the corresponding contour lines.

Linguist A. Korzybski emphasized the essential but often-forgotten distinction between things and the ways they are represented in his remark 'the map is not the territory'. Of course, the video is not the territory either.

You will find that the video reviews some of the topics you met in Section 2. But it also looks forward to the use of grid bearings and compass bearings to estimate position and direction. You will be meeting bearings in the next subsection.

You will probably find it useful to read through the next activity before you watch the video band and then complete the activity after viewing. Have paper, pen and your map with you as you watch; do not hesitate to stop, start or rewind the tape should you want to study a particular sequence closely. Make notes and sketches to help you tackle the activity. You may also wish to view the video band again as you complete the activity.

Activity 21 *Learning File: using the video*

Imagine you have been asked by the local scout leader to help a small group of young scouts prepare for their annual hill walk. The scout group is to stay in the Peak District and go on the walk shown in the video. The party leader is particularly keen to teach the scouts how to use a compass to take bearings and to help the scouts appreciate some of the land features, especially Hollins Cross. Imagine you have decided to accept the challenge and begin to prepare material you would use to teach the scouts:

(a) how to take a compass bearing;

(b) to recognize particular features from the map.

Outline and record the steps you would take in achieving these goals.

◇ What equipment including video material might you use?

◇ How would you deal with technical terms such as 'saddle point'?

◇ What are the main points you want to bring out?

The main features of the shape of a geographical saddle point are similar to those of a horse's saddle.

You may wish to return to this activity to review and perhaps enhance your notes when you have completed the rest of Section 3.

If you are able to, try to follow through this activity (or a similar one). When it is complete, note down whether you found 'explaining' a useful way to help your own learning.

Now watch band 5 on Videotape 1.

3.2 Measuring and predicting bearings

On the walk you will be using a compass to check your bearings and so navigate from point to point. A compass bearing is the direction given by a magnetic compass of one point measured from another, usually your own position. A traditional way of giving bearings and one used extensively in the past for navigation at sea was to use the four cardinal points of the compass: north, south, east and west subdivided by the directions north-east, south-east, south-west and north-west.

'Cardinal' here means 'of primary importance'.

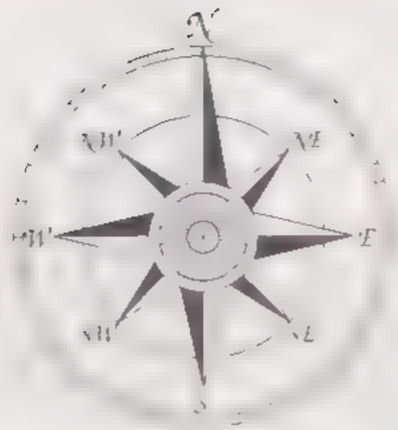


Figure 28 Named points of the compass

These directions can be further subdivided, to give a total of thirty-two named points of the compass. For example, between north and north-east lie the directions north-by-east, north-north-east and north-east-by-north. Accurately specifying a direction quickly becomes quite complicated! However, there is a different approach, which is to give a numerical bearing as the angle measured in degrees between north and the direction you are interested in. The angle is measured clockwise from north. So, as Figure 29 shows, a bearing of 90 degrees is the same as the result of turning through an angle of 90 degrees clockwise starting from north.

'To box the compass' means to name all thirty-two points of the compass in the correct order.

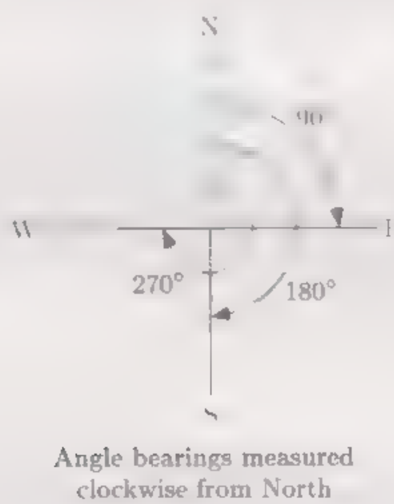


Figure 29 Compass bearings using angles

Thus, a bearing of 90 degrees corresponds to east. Similarly, bearings of 180 degrees and 270 degrees correspond to south and west, respectively. Because a full circle is divided into 360 degrees, a bearing of 360 degrees is the same as a bearing of 0 degrees, and corresponds to north.

Activity 22 Compass bearings

What are the bearings in degrees that correspond to the four compass directions north-east, south-east, south-west and north-west?

Measuring all angles clockwise from north is a convention among map-users. It is, however, not the same as the convention used by mathematicians for stating angles. Different groups of people do things in different ways—just as the customs of one country may differ from those of another. In any particular community, a convention involves the use of a common language to help communicate effectively with others.

The lines of longitude drawn on a globe run between the North Pole and the South Pole.

Another convention is that Ordnance Survey maps are drawn with north at the top. The northern direction of the vertical blue grid lines on your map is called 'grid north'. Because of the way an OS map is drawn as a flat representation of a curved surface, grid north differs slightly from 'true' north—that is, the direction of the North Pole. On your map, the difference is very small.

However, there is yet another north which is important to anyone using a magnetic compass and one which you do have to take into account. The north-seeking needle of a magnetic compass points to magnetic north, the geomagnetic pole in the Northern Hemisphere.

Like a bar magnet, the Earth has a magnetic field to which the needle of a magnetic compass responds. There are magnetic poles in the Northern and Southern hemispheres. The north magnetic pole is located geographically in the north of Canada, while the south magnetic pole lies in the Antarctic.

It is important to be clear about the difference between grid north and magnetic north. Grid north is not a place but a direction, one which has been fixed by the map-makers of the Ordnance Survey as a part of the National Grid reference system. The direction does not change with time. So, although features of the landscape of Great Britain may change, the National Grid remains as a fixed reference. Grid north, and the grid system generally, is a map phenomenon—something that exists only on an OS map.

The Earth's magnetic field, on the other hand, is a physical, changing phenomenon. It is something that can be detected in the real world. Magnetic north is the direction of the north magnetic pole, which can be located by devices which respond to the Earth's magnetic field. This is what a magnetic compass does.

Magnetic north is not subject to decisions taken by map-makers. Indeed, unlike grid north which is permanently fixed, the direction of magnetic north changes slowly over time owing to gradual changes of the Earth's magnetic field.

Records of the difference between true north and magnetic north have been kept for England since the late sixteenth century. In London in 1576, magnetic north was estimated to be over 10 degrees *east* of true north. Over the next 250 years, it drifted west by more than 30 degrees, reaching a maximum of nearly 25 degrees west in 1823. It is now drifting back towards the east.

If you look at the legend on your map, you will find some information about how grid north and magnetic north are related. In 1994, the direction of magnetic north was about 5 degrees west of grid north. This difference is not fixed but varies slowly with time. At the time of writing (1994), the difference between the direction of magnetic north and grid north is decreasing at a rate of about 0.5 degrees every three years.

Note that all this information is valid only for the region covered by your particular map. The direction of magnetic north varies with place as well as with time. So on a map of a different part of Great Britain or in another country, the angle between grid north and magnetic north may well be different.

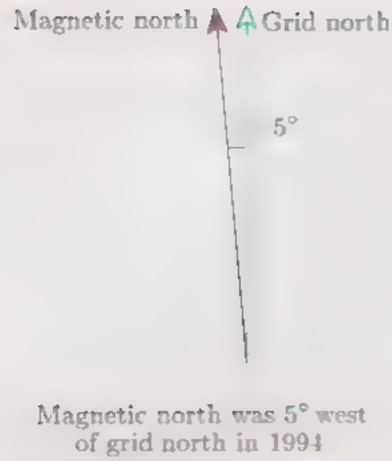


Figure 30 Magnetic north and grid north

Activity 23 *Predicting magnetic north*

In the region covered by your map, the direction of magnetic north is currently drifting towards grid north. If the current rate of drift is maintained, in what year will the directions of magnetic north and grid north coincide?

OS maps are revised and reprinted from time to time. If you can get hold of a more recently published map, check to find the most recent estimates of direction and rate of change of magnetic north. How do they compare with the predictions made in 1994?

Take stock for a moment. On the one hand, you have an OS map on which a grid has been overlaid. This grid has been defined by the map makers and anyone can use it to fix locations and to measure angles between different points on the map. When working with a map, you are working within a mathematically and geographically defined space, not the real world. The map is not the country.

The walk, on the other hand, *is* taking place in the real world. Places are real places and not grid references. And compass bearings give information about angles relative not to an imaginary grid but to a physical phenomenon: magnetic north.

An OS map is a complex symbol. But people often act as if the map were the country and do similar things on the map as they do in the real world. In the real world, you measure distances and take compass bearings. On the map, you can also measure distances and angles, and relate these measurements to physical reality: a distance measured on the map is directly proportional to the distance in the real world, an angle measured on the map is equal to an angle measured in the real world. This is not a happy accident; OS and other maps are drawn in special ways so you can

do just that. The map can be used as a model to predict what the distances or bearings are going to be when measured in the material world.

You saw in the last subsection how distances on the map and the ground were related. Now look at the relationship between bearings measured on the map and compass bearings.

If you are unsure how to use a protractor to measure angles, work through the audio sequence 2 of Audiotape 2 now. The frames accompanying this band are to be found in the Appendix on page 105.

A bearing measured on a map relative to grid north is called a grid bearing. Grid bearings are measured clockwise starting from grid north, and therefore use the same convention as compass bearings.

Activity 24 Bearing A from B

From Mam Tor, you can see the village of Nether Booth (grid reference 142861) in the Vale of Edale. Calculate the grid bearing of Nether Booth from Mam Tor as follows.

Using a pencil and ruler, draw a straight line on your map connecting Mam Tor and Nether Booth. Now draw a line through Mam Tor in the direction of grid north, parallel to the north–south grid lines. Figure 31 is a sketch of the bearing. Using a protractor, measure the angle between grid north and the line to Nether Booth.

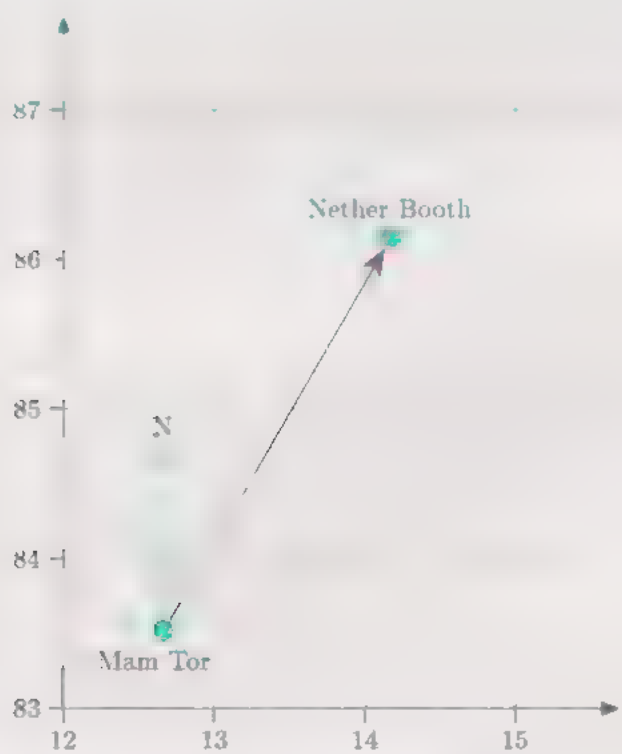


Figure 31 Bearing of Nether Booth from Mam Tor

You should have found that the grid bearing of Nether Booth from Mam Tor is about 30 degrees.

Now what would happen if you actually stood on Mam Tor and used a compass to take a bearing on Nether Booth? You would get a reading of about 35 degrees, five degrees greater than the one measured with a protractor on the map. Why? Because the compass bearing gives the angle between *magnetic north* and the direction of Nether Booth. And, since (in 1994) magnetic north lies some 5 degrees to the west of grid north, the compass bearing is 5 degrees *greater* than what you would measure on the map. Figure 32 shows how this works.

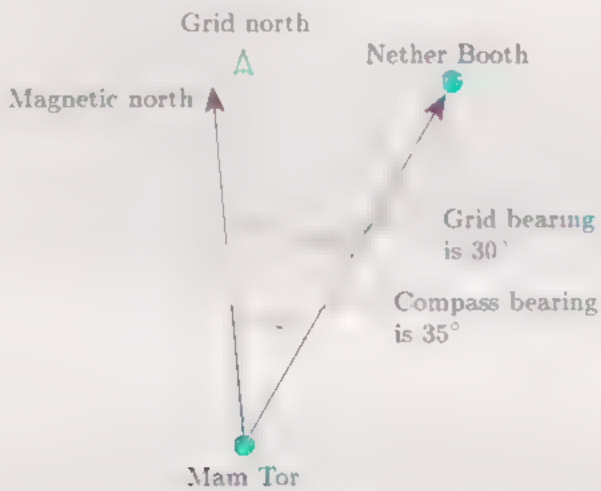


Figure 32 Grid and compass bearing of Nether Booth from Mam Tor

So there is something to remember when using a compass and an OS map. To convert grid bearings to compass bearings, just add the difference between grid north and magnetic north. To go the other way, and convert compass bearings to grid bearings, just subtract the difference. Among walkers there are several mnemonics used to help remember whether to add or subtract. These include 'add for mag, get rid for grid' and 'the ground is bigger than the map, so the compass bearing is bigger than the map bearing', or you can make one up for yourself such as 'From mAp to grounD: ADd; from gRound to map: Remove'.

Of course, this rule will have to change in the UK as magnetic north moves to the east of grid north. But this will probably not happen until after about 2024.

Activity 25 Checking directions

On the video, you saw the walkers check the direction of the path at a point (grid reference 132839) near Mam Farm. Use your map to find the grid bearing and hence predict the compass bearing from this point to the junction between the path and the road (grid reference 128831). Can this junction be seen from where the walkers were standing?

On walks in clear weather, where a compass is used to take a bearing on a landmark such as a hilltop, the difference between grid north and magnetic

north is unlikely to cause any major problems in navigation. However, if you are crossing open country, or accurate bearings are crucial for your safety, it is important to take the difference into account.

It is also important to be aware of the effects of errors in bearings. Errors up to about 5 degrees are not unusual, even among experienced walkers, and can occur at various stages: reading a bearing from a map, correcting for magnetic variation, following a bearing. Errors can be compounded so that a walker might end up following a bearing that may be out by 10 or 12 degrees. Over a kilometre, a 12-degree error would result in a deviation of over 200 metres from the required path.

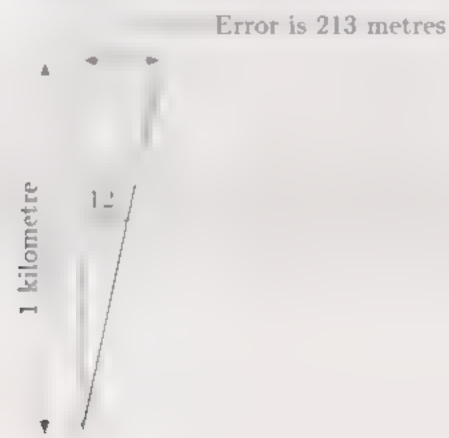


Figure 33 Effect of errors in bearing

If you are walking in open country, you should be able to check your own position using your map and a compass.

Activity 26 *Predicting compass bearings*

Suppose you are planning a walk in the High Peak on Blackden Moor, and want to check your position when you have reached the part of the path which has grid reference 113888, near the area labelled Seal Stones.

Using your map, predict the compass bearings of Upper House Farm (119899) across the valley and of Blackden View Farm (132896), further to the east, from your desired location 113888.

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Now think about Activity 26 the other way around. What would be the compass bearing of the location 113888 from each farm? Figure 34 shows that the reverse bearings are always just the original bearings plus 180 degrees. This is true whether you are using compass bearings or grid bearings. So looking back from Upper House Farm the compass bearing will be $33 + 180 = 213$ degrees, while from Blackden View Farm the compass bearing will be $72 + 180 = 252$ degrees.

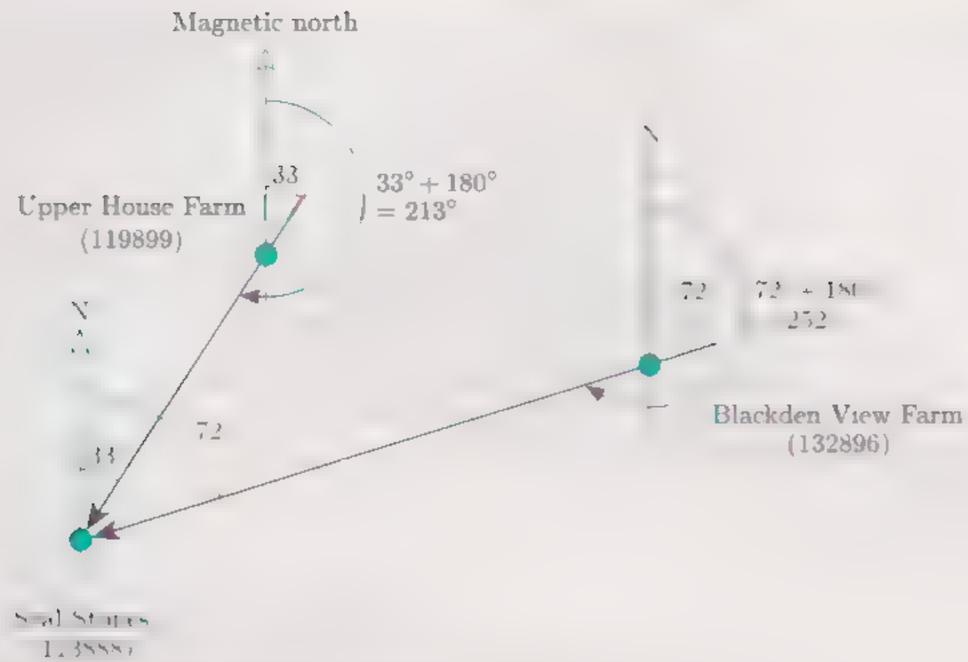


Figure 34 Reverse compass bearings

Two bearings are enough to fix your location on Blackden Moor. But what if the bearings you measure are not the ones you expect?

Activity 27 Where am I?

Suppose you get compass bearings of 18 degrees for Upper House Farm and 61 degrees for Blackden View Farm. Where are you?

In Activities 26 and 27, the reverse compass bearings were less than 360 degrees. But what would it mean if the result of adding 180 degrees to a bearing came to more than 360 degrees? Suppose you started with a bearing of 230 degrees. Adding 180 degrees to this gives 410 degrees. But a bearing of 360 degrees is the same as a bearing of 0, so 410 degrees corresponds to 410 − 360 = 50 degrees. But notice that you also get 50 degrees if you *subtract* 180 degrees from 230 degrees. Try these computations for a few angles greater than 180 degrees: you will find that it works every time. So here is a general mathematical rule: if a bearing is greater than 180 degrees, subtracting 180 degrees always gives the same answer as adding 180 degrees first, and then subtracting 360 degrees. Why?

Finding the position of a third point based on the bearings taken from two others forms the basis of a technique called *triangulation*, used in surveying.

Activity 28 Why does it work?

Spend a few moments thinking about this rule. Make a few notes for yourself to explain why the rule works. You may find a diagram helps.

In the National Grid system, an easting on its own tells you that the location you want lies less than 100 metres to the east of a particular north–south line. The accompanying northing says that the location lies less than 100 metres to the north of a particular east–west line. The location lies within the 100-metre square where the lines cross.

Similarly, you need two separate bearings to specify any particular point. Each bearing tells you only that the point lies somewhere along a line in a particular direction. Where the two lines cross both items of information are brought together to specify the location exactly.

When you use bearings to determine a location, you are using mathematical ideas similar to those in a grid reference system. A map is a two-dimensional surface. In other words, any point on a map can be located by just two independent items of information. On an OS map, the items of information may be a pair of bearings, or an easting and a northing; on a map in a world atlas, the information may be latitude and longitude; on a street plan, the square in which a particular road may be found may be specified by a letter and a number. Once you understand how one system works you can transfer your knowledge to others—all you need are the features of the particular system you are going to use.

Remember, however, that any reference or bearing must be measured with respect to a fixed point somewhere: an easting or northing is a distance measured relative to the south-western corner of the relevant 100-kilometre square; a bearing is an angle at a given point measured relative to grid north or magnetic north.

To finish off this subsection, return to the walk. On the video you saw the walkers use their compass at Lose Hill to check the direction of the path to Losehill Farm.

Activity 29 Journey's end

Using your OS map and a protractor, predict the compass bearing of Losehill Farm from the top of Lose Hill.

3.3 Time for a walk

It is useful to be able to estimate the time that a walk will take. Walkers may need to catch a bus or a train at the end of the walk, or to meet a car at a pick-up point. They can also plan when to take breaks—and make sure that there is enough time to complete the walk in the daylight. It is always a good idea for walkers to let someone know where they are going, and how long they expect to be out.

A well-known rule of thumb, called Naismith's rule, is used by walkers to estimate the time taken to walk across open, hilly country.

Historical note

William Naismith (1856–1935) was a Scottish climber and alpinist. He formulated his rule for estimating hill walking times in 1892. The original rule assumed a walking speed of three miles per hour, plus an extra thirty minutes for every thousand feet of ascent.

Naismith's rule is based on experience and has two parts: one is concerned with an average walking speed over flat ground and the other with the extra time it takes to climb up any slopes. In the modern form of the rule, the walking speed is expressed in kilometres per hour and the height of ascent in metres. It can be stated as follows.

The time required for a walk in open hilly country can be estimated by:

(a) assuming an average speed of five kilometres per hour;

(b) adding thirty minutes for every three hundred metres of climbing.

Notice that Naismith's rule does not include any extra time for coming down a slope, which seems not to take up as much time as going up, unless of course the slope is particularly steep and you have to pick your way carefully.

To estimate the time to complete a walk, you need to know how long the walk is in kilometres and how much height you expect to gain (adding each ascent, not just looking at total height gained overall).

Activity 30 Word formula

Stop for a moment and think about how you would write Naismith's rule as a word formula.

How would you complete the following?

> The estimated time for a walk is equal to . . . .

One way of looking at Naismith's rule is to see it as a way of splitting the walk into two parts: a horizontal part and a vertical part. The total travel time is then calculated by working out and adding the times for two separate journeys: one covering the horizontal distance at a speed of 5 kilometres per hour and the other covering the vertical distance at a speed of 600 metres per hour.

Ignore first of all any change in height and deal just with the average, or mean, walking time. Naismith's rule assumes an average walking speed of 5 kilometres per hour on reasonably level ground, so write:

> the walking time in hours is equal to the distance in kilometres divided by the average speed of 5 kilometres per hour.

Or, rather more concisely,

$$\text{walking time (in hours)} = \frac{\text{distance in kilometres}}{\text{average speed in kilometres per hour}}$$

$$= \frac{\text{distance in kilometres}}{5}$$

In other words, the assumption is that a walker can travel the *horizontal* distance between two points at a speed of 5 kilometres per hour. For example, a 10-kilometre walk over reasonably level ground should take about $10/5 = 2$ hours.

Now include the information about climbing. Note first how the height change is calculated. Figure 35 shows the profile of a path which starts at a height of 50 metres, climbs to 80 metres and then descends to 40 metres before climbing again to 110 metres. Although the overall change in height from 50 metres to 110 metres is 60 metres, Naismith's rule requires the individual climbs to be added together, giving a total of $30 + 70 = 100$ metres.

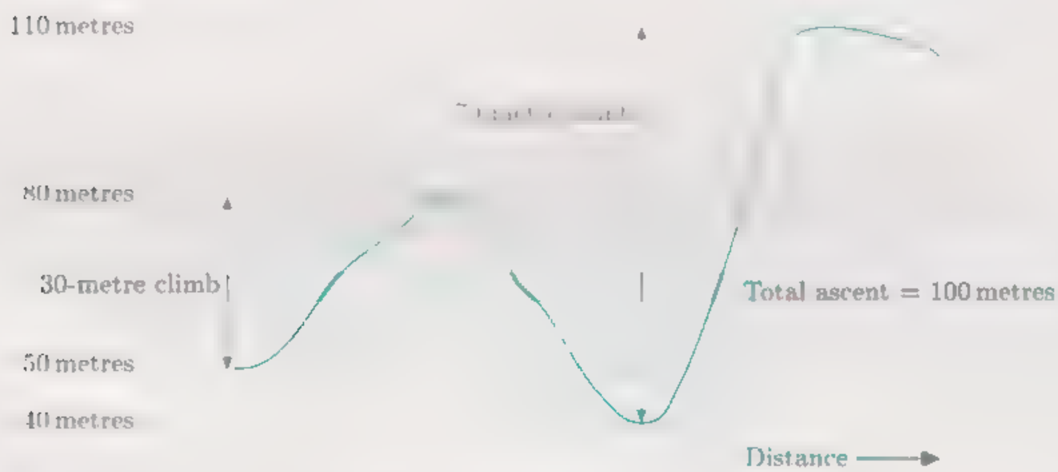


Figure 35 Calculating height for Naismith's rule

The assumption, based on Naismith's experience, is that if the walk involves an ascent then a climb of 300 metres will add an extra 30 minutes, or 0.5 hours to the time. Another way of thinking about it is that a climb of 10 metres, or the distance from one contour line to the next on an OS map, adds about 1 minute to the estimated walking time.

On this basis, an ascent of 600 metres will add 1 hour to a walk. The assumption built into Naismith's rule is that a walker travels the *vertical* distance at a speed of 600 metres per hour.

In other words, the second part of the word formula is:

> the extra climbing time in hours is equal to the total ascent in metres divided by a vertical speed of 600 metres per hour.

Or,

$$\text{extra climbing time (in hours)} = \frac{\text{total ascent in metres}}{\text{vertical speed in metres per hour}}$$

$$= \frac{\text{total ascent in metres}}{600}$$

So, if a walk involves a total climb of 250 metres, it will add about $250/600 = 0.42$ hours, or 0.42×60 minutes—that is, about 25 minutes—to the walk.

Naismith's rule puts the walking time and the extra climbing time together in one formula to give an estimate in hours of the total time for a walk.

Time taken for walk

= walking time + extra climbing time.

Or,

time taken for walk (in hours)

$$= \frac{\text{distance in kilometres}}{5} + \frac{\text{total ascent in metres}}{600}$$

Notice that both parts of this formula have hours as the units of time, even though the unit of length is kilometres in the first part, and metres in the second part. This is because the average walking speed is expressed in *kilometres* per hour, while the average climbing speed is expressed in *metres* per hour. Dividing distance in kilometres by an average walking speed in kilometres per hour gives an answer in hours, as does dividing the total ascent in metres by an average climbing speed in metres per hour. This is an important point to appreciate. Physical quantities such as times or distances or speeds are not just numbers; they are always associated with units. When such quantities are added or subtracted the units must be the same for each quantity. It is meaningless, for example, to add 5 kilometres to 100 metres to get 105. The sum makes sense only if both are expressed in kilometres: $5 + 0.1 = 5.1$ kilometres, or if both are expressed in metres: $5000 + 100 = 5100$ metres.

Activity 31 Using Naismith's rule

The first part of the walk from near Mam Farm to the summit of Mam Tor is about 2 kilometres long, and over this distance the ground rises from about 300 metres to 517 metres.

Use Naismith's rule to estimate how long this section will take.

Underlying Naismith's rule is a *model*, a formulation about how walks will go in general. As you have already seen, models offer the possibility of prediction ahead of time. By using some specific information about the

walk you intend to make plus some general assumptions about the speed at which a walker can travel across open hilly country, Naismith's rule provides an estimate of how long the walk will take. It is a *mathematical* model because it uses mathematics to predict the time, instead of you doing the walk to find out.

You will come across other mathematical models in this course. All such models use assumptions about the situation they represent. They stress some features and ignore others. Stop for a moment and think about the assumptions underlying Naismith's rule.

Activity 32 What are some assumptions?

Think carefully about Naismith's rule.

- What do you think Naismith's rule stresses and what does it ignore about hill walking? Note down as many ideas as you can think of.
- Where do you think the figures for walking and climbing speed come from?

Naismith's rule is based on experience gained from a lot of walking over different types of country. The average walking and climbing speeds have been chosen so that the rule gives a fair estimate of time for reasonably experienced hill walkers. The model is therefore directed at the activity of a particular group of people. Naismith's rule is less likely to give reasonable predictions for a group of young children on a school walk, or for Army paratroopers on an exercise march.

Of course, what is reasonable for hill walkers is a matter of judgement, but if the predictions were such that the rule consistently gave times which were, say, thirty minutes too short or too long, then you might want to think about revising the values of the average speeds in the model. The average walking and climbing speeds used in Naismith's rule are called the *parameters* of the model. The values of the parameters are not sacrosanct, but walkers have found that the rule gives useful and reasonable estimates of time.

Naismith's rule is a planning tool which provides the means to plan a walk to fit the time available. It is wise to allow a safety margin of around 25% in your time estimates. So if Naismith's rule indicates that a walk will take about six hours, then allow seven and a half to eight hours to complete the journey.

Thinking about the assumptions underlying a model is important when you use mathematics and when you are applying the model. Sometimes the assumptions do not fit a particular situation, and you may have to make some adjustments. You may want to question how well Naismith's rule fits the circumstances: how physically fit are the walkers? Are they going to tire quickly? Will the terrain make walking particularly difficult? The answers to these questions will help you to use Naismith's rule effectively.

Activity 33 Relationships and rules

Naismith's rule is an example of a useful rule of thumb. You may have come across other such rules at home or at work. For example, estimating the amount of paint or wallpaper to buy to decorate a particular size of room, estimating the amount of food to buy for a meal for a number of people, or estimating cooking times for different types of food. You may also know about rules which are linked to specific jobs at work.

Take some time now to think about what rules and relationships you use, or have come across. They do not have to be rules of thumb – they can be hard and fast like the way your electricity bill or telephone bill is [illegible] you would explain it to someone else.

Recall Looking with relationships in *Unit 1*

Where did the rules and relationships come from? Note down your thoughts with your answer.

To finish this subsection, use Naismith's rule to estimate the time for the whole walk from Mam Farm to Losehill Farm. If you were just about to set out on this walk it would be a good idea to gather all the information about places, grid references, distances, compass bearings and times together on a handy reference card, called a route card, that you can take with you. For safety, you could give a copy of your route card to someone who was not coming with you, to let them know where you were planning to go, and how long you would be out.

Route cards do not have to be elaborate affairs, but they should contain all the basic planning information to help you on your walk. Figure 36 shows a route card partially completed for this walk.

Date	11 April 1995				
From: Mam Farm		**grid ref:** 133840		**starting time:** [illegible] am	
To: Losehill Farm		**grid ref:** 158846		**est. arrival time:**	
Path to	Grid reference	Compass bearing (degrees)	Distance (kilometres)	Height climbed (metres)	Estimated time (minutes)
A625 road	128831	212	1.13	90	[illegible]
Footpath	125831	287	0.3	20	[illegible]
Mam Tor	127836	352	0.6	107	[illegible]
Hollins Cross	136845	24	1.33		[illegible]
Back Tor	145849	91	1.08	40	19
Lose Hill	153853	35	0.88		
Losehill Farm	158846	156	0.85		
Total			6.17		

Figure 36 A route card for the walk

At the top of the route card is basic information about the starting and finishing points of the walk, your planned starting time and your estimated time of arrival.

The first column gives the location you are aiming for on each stage of the walk, and the second column gives each location's grid reference. Next comes the compass bearings. These have been predicted from the map by measuring the grid bearing of the path from each location to the next, and then adding 5 degrees. For example, at Mam Farm, the predicted compass bearing of the path to the A625 road is 212 degrees.

The fourth column gives the distance of each stage in kilometres, again predicted from the OS map. Into column five goes the the total height climbed in metres at each stage. Remember that Naismith's rule uses the total ascent in metres, but takes no special account of any descents. Finally column six gives the estimated time for each leg of the walk, calculated using Naismith's rule.

Here are some sample calculations. The distance from Mam Tor to Hollins Cross is 1.3 kilometres—there is no climb to take into account because the path drops all the way. Assuming a walking speed of 5 kilometres per hour, Naismith's rule gives the estimated time as 1.3/5 hours, or $1.3/5 \times 60$ minutes or about 16 minutes.

Back Tor is 1.08 kilometres on from Hollins Cross. On this stage, the path rises slightly to follow the 400-metre contour, falling back to a height of 391 metres as it approaches Back Tor. There is then a sharp rise of about 30 metres up the side of the Tor. According to Naismith's rule the time from Hollins Cross will be $1.08/5 = 0.22$ hours, or about thirteen minutes, plus an extra four minutes for the overall 40-metre climb. As you saw on the video, however, Back Tor is a short but an extremely steep climb and you may want to allow a couple of minutes extra to get up it. Allowing six minutes, therefore, for the total ascent between Hollins Cross and the top of Back Tor gives a total time for this stage of nineteen minutes.

Activity 34 Completing the route card

Now complete the route card for the last two stages of the walk.

Use your OS map to find the height data for the last two stages of the walk. Then use Naismith's rule to predict the time they will take.

Estimate the time for the whole walk, and fill in on the route card your expected time of arrival at Losehill Farm.

You can check your estimate by adding up the height climbed at each stage to find your total ascent, and then using Naismith's rule on the complete walk.

This section has been concerned with visualizing the information contained on a map, working with angles and bearings and using a mathematical model of a walk to make predictions.

Subsection 3.1 drew on a video band to show different perspectives of the planned walk: one was that of the walkers. Decisions about which route to take or which path to follow were made with the help of the Ordnance Survey map and a compass. An understanding of the conventions used on the map and on the compass were necessary to interpret the information correctly. Learning to recognize, read and understand different conventions for representing information and ideas is an important mathematical skill.

Another perspective was provided by computer-generated graphics. Using the information given by the contour lines, a three-dimensional representation of the landscape was built up. From this view you were able to see how the patterns of the contour lines are interpreted in terms of the shapes of the hills and valleys.

Subsection 3.2 looked at angles and direction bearings. In map work, angles are measured in degrees clockwise from north. This is just one particular convention and you will meet other ways of specifying angles as you work through the course. Comparing compass bearings and grid bearings involves a correction for the difference between magnetic north and grid north.

Bearings taken on landmarks can be used to check position. On a two-dimensional surface using a fixed reference system, such as a map, only two numbers are needed to specify a given location. These numbers may be an easting and northing pair of the National Grid system, or two compass bearings.

Subsection 3.3 discussed Naismith's rule—a rule of thumb used by walkers and climbers to estimate the time for a walk over open hilly terrain. Naismith's rule expresses in words a mathematical relationship between the distance walked, the height climbed and the time taken for the walk. It not only provides a simple mathematical model of a walk, based on experience, but also provides useful estimates of time when the assumptions on which it is based match the conditions of the walk and the walkers. Try to think about what other mathematical rules you have come across at home or at work.

The section ended by completing the plan of the walk and drawing up a route card. Before completing this section, take a few minutes to review your work: complete the Learning File activities and check the outcomes. Pausing to review and reflect is an important part of learning. Do you remember the model of learning that was described in *Unit 1*? Reviewing your learning can be thought of as the continual cycling back and forth between experience (the learning you have been involved in) and reflection. Look back to your response to Activity 5 in this unit. Add notes relating to the particular techniques covered in this section (use the outcomes list to help here). Now that you have gained considerable experience using OS maps what can you 'see' in them? Add your comments to your response to Activity 2.

Outcomes

After studying this section, you should:

◇ feel more confident about making notes from a videotape, with the aim of giving an accurate description to others (Activity 21);

◇ be aware that every mathematical model stresses some features of the real world, and ignores others, and that all models carry assumptions (Activity 32).

You should also be able to:

◇ use the following terms accurately and be able to explain them to someone not taking this course: 'grid bearing', 'compass bearing', 'grid north', 'magnetic north', 'mathematical model' (Activities 22 and 23);

◇ use a protractor to measure grid bearings an an OS map (Activities 24, 25, 26, 29 and 34);

◇ predict a compass bearing given a grid bearing and information about magnetic north, and vice versa (Activities 25, 26, 29, 31);

◇ fix a location using the bearings from two other points (Activity 27);

◇ say in words, and use a word formula to express a relationship, as in Naismith's rule (Activities 30, 31, 33, 34).

4 Slopes and sizes

Aims The main aim of this section is to explore the mathematics embedded in the calculation of slopes and areas, and to introduce line graphs. ◇

You should now be familiar with the idea that the spacing of contour lines on an Ordnance Survey map gives an indication of the steepness of the ground. In this section, you will see how mathematics is used to be more precise about the slope of a hillside, or the gradient of a road. You will learn how height and distance information from a map can be presented as a graph, giving a different visual representation of gradient.

Recall 'Looking with graphs and diagrams' in *Unit 1*.

An OS map contains more than height and distance information, however. It also can be used to predict the size of actual areas as well as distances. Think back to the method you used to convert distances on the map to actual distances on the ground. An area on the map is related to the corresponding area on the ground by a scaling factor. The area scaling factor is not usually stated explicitly, but it can be calculated from the map scale (which tells about *lengths*). Areas on the map can be thought of as representations of areas on the ground.

Calculations of distances, heights and areas are calculations which involve units of measurement, as well as numbers. This section includes a brief look at the SI system of units of measurement used in the mathematical, technological and scientific communities.

You will need your OS map for parts of this section.

4.1 Picturing steepness

In Subsection 2.3 and the video band 'Getting your bearings' which you watched in Subsection 3.1 you saw in fairly general terms how contour lines are used to represent the three-dimensional shape of the landscape, and also how the spacing of the contour lines gives information about slopes. Now look at how mathematics is used to define the steepness of a slope in terms of a number.

The steepness of a slope is called its *gradient*. From a mathematical point of view, the gradient expresses a relationship between changes in height and distance. The average gradient of a slope between two points is defined as the change in height divided by the horizontal distance, as shown in Figure 37. If you are working from a map, the change in height can be found from the contour lines and the horizontal distance over which the change takes place is found by using the map scale (you could also, at times, calculate changes in height using spot heights).

Figure 37 Definition of average gradient

Here is an example from the walk. The map shows that the height of the path from Back Tor to Lose Hill dips to 410 metres about 370 metres from the top of Back Tor. From there the path rises about 66 metres over a distance of roughly 500 metres as it climbs to the top of Lose Hill.

▶ How steep is the path?

The average gradient is defined as:

$$\text{gradient} = \frac{\text{change of height}}{\text{horizontal distance}}$$

which, in this case, is:

$$\text{gradient} = \frac{66\text{ metres}}{500\text{ metres}} = 0.13.$$

This is interpreted to mean that the ground rises 0.13 metres (13 cm) for every metre moved horizontally: a reasonable slope to walk up.

If both the change of height and the horizontal distance are measured in metres, the gradient relates metres measured vertically to metres measured horizontally. But what would be the gradient if you measured height and distance in some other units?

▶ Would the gradient be a different number if you measured in feet or yards, or even in an ancient unit such as the cubit?

A *cubit* was an ancient unit of length based on the length of the adult forearm from the elbow to the tip of the middle finger.

If the height and distance measurements use the same units—whatever they are—the number associated with the gradient stays the same. This is because the gradient is a ratio of the height to the distance and, as long as both measurements are made in the same units, their ratio will be the same.

Activity 35 *Finding gradients*

On the north-west slope leading up to Hollins Cross, the ground rises by 100 metres over a horizontal distance of about 300 metres.

(a) What is the gradient of the slope, using measurements in metres?

(b) Use the approximate equivalence that there are 3.281 feet in 1 metre. Calculate the gradient using measurements in feet.

(c) Use the approximate equivalence that 1 cubit is 0.49 metres. Express the height and distance in cubits and work out the gradient.

The point to remember is that the gradient relates a vertical distance to a horizontal distance. It does not matter what the units of measurement are: the gradient here is simply a number which relates units of the same type. For a particular slope, the number is the same irrespective of the units.

▶ Can you give another example where a pure number is used to relate two measurements of length?

You should recognize the idea from the discussion in Subsection 2.2 about map scales. Recall that the map scale relationship was:

distance on the ground = map scale × distance on the map

where the map scale is a number without any units, and the distances on the ground and on the map are measured in the same units as each other.

The relationship between height and distance can be written in a similar way:

change in height = gradient × horizontal distance

where the change in height and the horizontal distance are in the same units, and the gradient is simply a number.

Suppose you were on the top of Mam Tor. The slope to the east is very steep indeed, too steep for a path.

▶ What is the average gradient of the slope on this side of the Tor over 500 m?

Over a distance of 500 metres the height of the ground falls by 207 metres. This is a rather different situation from the previous case because the height is decreasing rather than increasing with distance. The convention is to think of the decrease as a negative increase, in the same way as taking money out of your bank account could be thought of as a negative change (negatively increasing) in your deposit. So the change is height is written as −207 metres in this case and the calculation is:

$$\text{gradient} = \frac{-207}{500} = -0.41 \text{ (to two decimal places).}$$

The negative gradient is interpreted to mean that, on this side of Mam Tor, the ground falls by just over 0.4 metres (40 centimetres) for every metre of horizontal distance.

Now return to the path from Back Tor to Lose Hill. Locate Back Tor on your OS map. At the top of Back Tor the path reaches a height of almost 430 metres, dipping to about 410 metres before climbing Lose Hill.

One way to get a feel for gradient is to make a sketch graph showing how the height of the ground changes as you move away from a particular point. By measuring along the line of the path on the map from Back Tor as best you can, you can estimate the distances from the Tor to where the path cuts the contour lines at 410 metres, 420 metres, 430 metres, and so on, to the 476 metre spot height at the top of Lose Hill.

One person's measurements are given in Table 3. The table gives heights along the straight line between Back Tor and Lose Hill.

Table 3 Distances and heights from Back Tor to Lose Hill

Distance from Back Tor (in metres)	Height (in metres)
0	420
83	430
170	420
370	410
500	420
600	430
670	440
720	450
750	460
820	470
870	476

Plotting this information as a sketch graph gives a representation of how the height changes with distance.

The word 'axes' is the plural of axis.

The frame of the graph is provided by two lines called *axes*. Figure 38 shows the horizontal axis running across the page and the vertical axis running up the page. The angle between the axes is 90 degrees; mathematically they are said to be *perpendicular* to each other.

The horizontal axis shows the numbers 0, 100, 200, 300, ..., 900 spaced at equal intervals. The axis has a label which says what the numbers represent, and the units they are measured in. The numbers represent the distance in metres along the path from Back Tor to Lose Hill.

The vertical axis shows equally spaced marks representing the numbers 400, 410, ..., 480. The label on this axis indicates that the numbers represent the height of the ground, and are in units of metres. A practical point to note here is that when you choose the scale on the axes of a graph, choose numbers that are convenient and easy to use with the data that you have. In Table 3, the heights are listed every 10 metres (except for the last value), so it is straightforward to plot them using the vertical scale in Figure 38.

Feint horizontal and vertical lines complete the grid frame of the graph. Finally, the graph needs a title, so you and others know what it claims to represent.

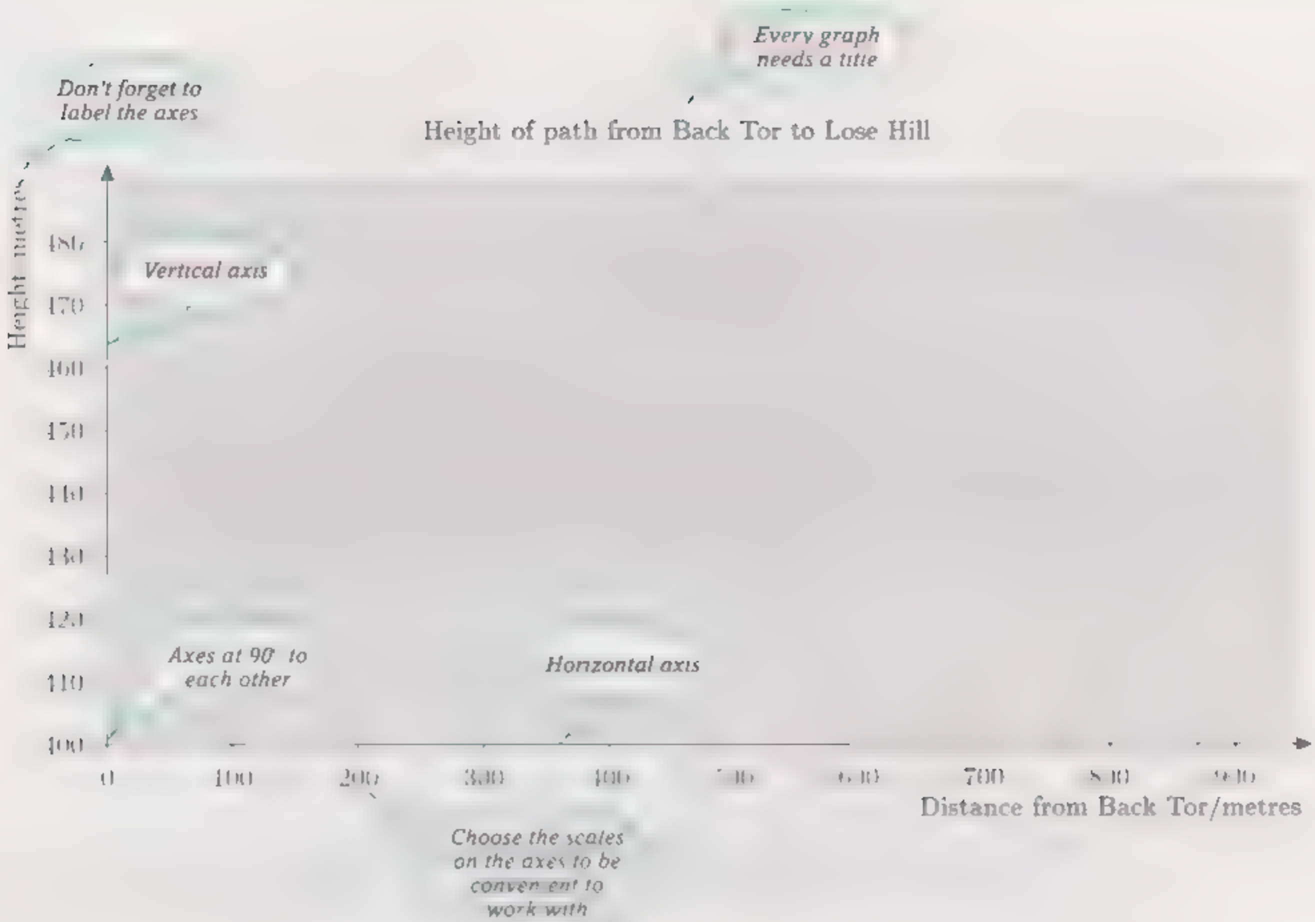


Figure 38 Setting up a graph

Figure 39 (overleaf) shows the same axes, but this time with some of the information plotted. Each point on the graph comes from a pair of numbers. The first number is the distance along the path, and the second number is the height of the ground. For example, the table shows that at a distance of 500 metres from Back Tor the height of the path is 420 metres. By convention in mathematics this point is written as (500, 420) — a pair of numbers separated by a comma and enclosed in round brackets. The numbers 500 and 420 are called the *coordinates* of the point, and the notation (500, 420) is called a *coordinate pair*.

Notice that this is not quite the same convention as that used for quoting Ordnance Survey grid references where the coordinates are given in the same order but without the comma and brackets. Remember that in a grid reference the easting and the northing always have the same number of digits, so there is no confusion about where a particular digit belongs. In mathematics, however, the two coordinates may well not have the same number of digits, so the comma is used to make sure that each coordinate is distinct and separate. The mathematical notation allows greater flexibility in using coordinate pairs where the numbers of digits in each coordinate are not always the same. Also, in mathematics, a coordinate pair refers to a specific *point* on a graph, and not to a particular *square region*, as in OS grid references.

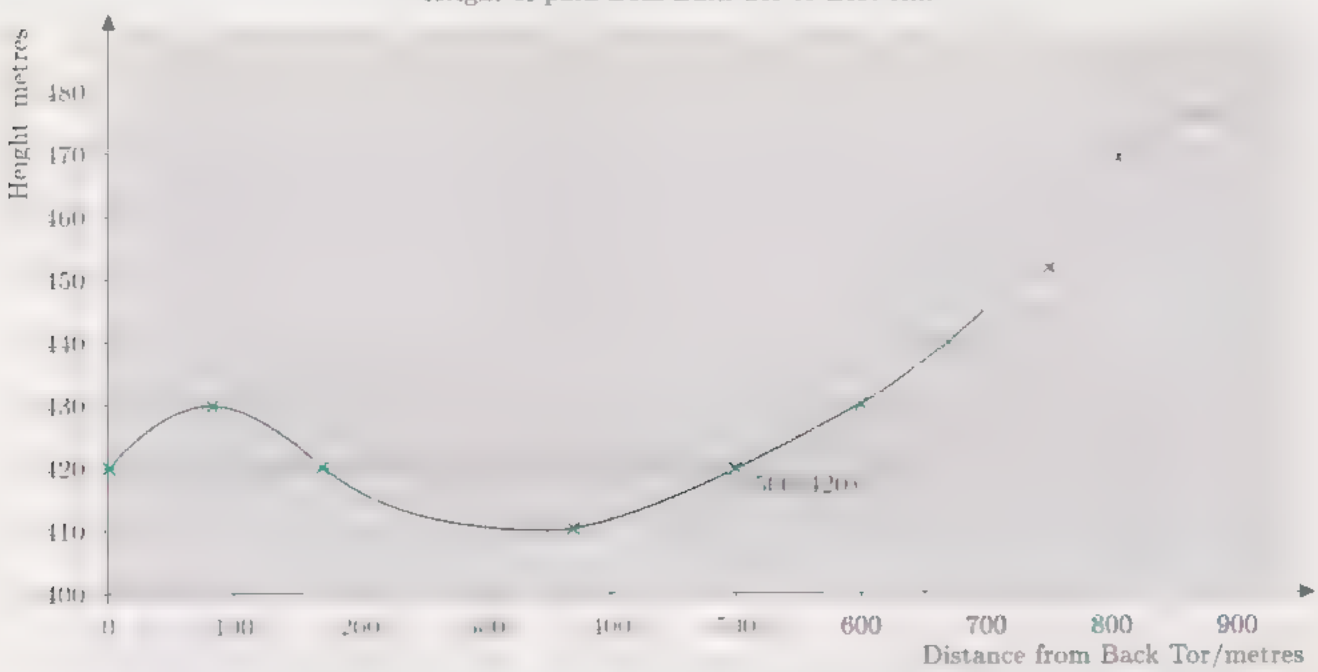


Figure 39 Graph of height plotted against distance

Activity 36 *Plotting points*

Plot and join up, using a smooth curve, the remaining points from Table 3 on Figure 39 to complete the graph of height against distance.

The small crosses are joined by a smooth line producing a *graph*. When joining up the points on a graph it is important to think about what the line itself represents. The line is a model which, in this case, represents the profile of the ground. Drawing a smooth line between the points suggests that the height of the ground changes smoothly from one point to the next. Be aware, though, that this is just an assumption: you have no information about what the ground surface is really like. All you have are the distinct, discrete coordinate points.

In any graph drawn from a set of coordinates, the line itself reflects the assumptions of whoever drew the graph. Next time you see a line graph, ask yourself why the line connecting the points (if there is one) is the shape it is.

The points themselves come from your OS map: the graph re-presents the map data in a new way, providing a visual way of expressing the relationship between height and distance. Like the map, the graph is not a simple picture of the profile of the ground but a symbolic representation which you can learn to read.

Activity 37 *Reading in and reading out*

Using the graph, how would you describe precisely in words how the height of the ground changes as you move from Back Tor to Lose Hill?

Recall from earlier, the figure of 0.13 for the gradient of the path up Lose Hill, based on an overall height rise of 66 metres over a distance of 500 metres. This is the average gradient of the slope, shown on Figure 40 by the dashed line. The calculation of the average gradient produces a numerical value: it is a mathematical model of the slope which stresses the change of height and the horizontal distance, but ignores all other details. The model does not predict the slope of any particular part of the path, but rather gives an expectation about its general degree of steepness.

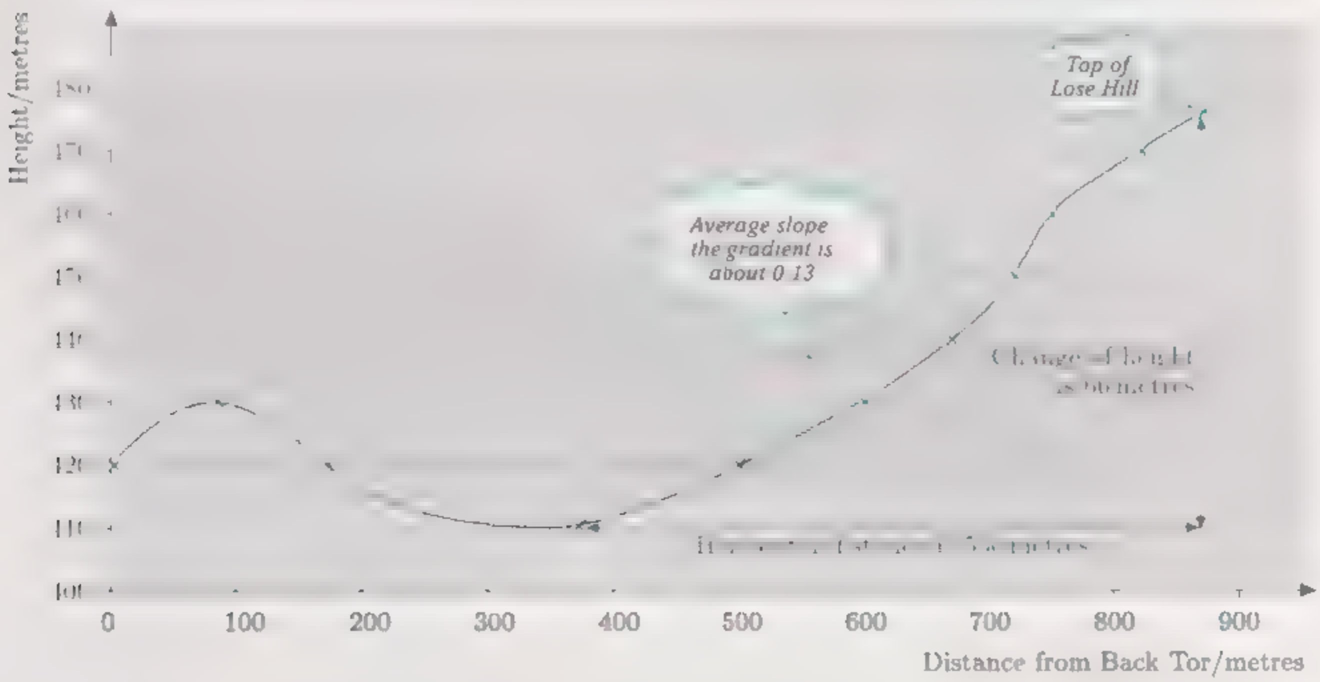


Figure 40 Completed graph of height plotted against distance

Average gradients, however, can be misleading where the landscape is less regular. For example, use the same technique at Mam Tor to look at the profile of the ground to the east. Putting information from the map about heights and distances results in Figure 41.

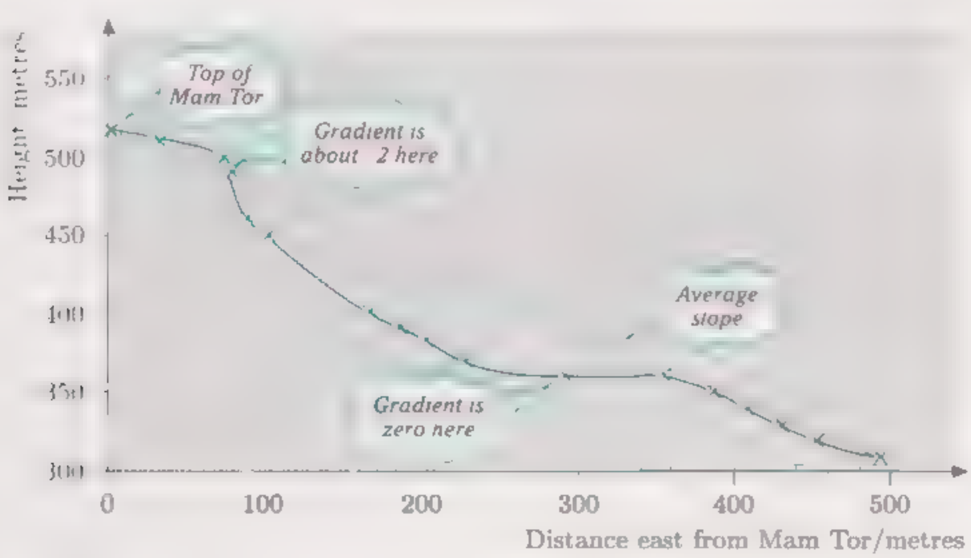


Figure 41 Profile of land to the east of MamTor

In this case, horizontal distances are measured in metres due east from the triangulation pillar at the top of the Tor. Notice that the scale on the horizontal axis of the graph runs from 0 (corresponding to the top of Mam Tor) to 500 metres. The heights are given directly in metres. The scale on the vertical axis runs from 300 metres to 550 metres, since the height changes by more than 200 metres over the 500 m distance. Had the scale on the vertical axis run from 0 to 550 metres, the result would have been a graph containing a lot of blank space, with all the interesting features near the top.

Always label axes so that you and others know what they represent, and what units have been used.

The horizontal and vertical scales on a graph are independent of each other. You choose them so that your graph shows clearly the features you want it to show, and it fits into the space you have available for it. In Figure 41, the scale on the horizontal axis was chosen so that five equal divisions, each representing 100 metres, would fit comfortably across the page. On the vertical axis, the scale was chosen to focus on just the heights between 300 and 550 metres, with equal divisions representing equal changes in height. The important point is that, whatever scales you choose, the axes of a graph are clearly labelled to avoid misunderstanding.

Activity 38 Descriptions

Look carefully at Figure 41. Describe in your own words and as precisely as you can how the slope of the land to the east of Mam Tor changes as you move away from the summit. How does it compare with the average slope?

Recall that the route of the walk follows the path along the ridge from Mam Tor to Hollins Cross.

Table 4 Distance and heights from Mam Tor to Hollins Cross

[illegible]	[illegible]	[illegible]
0	517	Summit of Mam Tor
63	510	
163	500	
263	490	
338	480	
375	470	
475	460	
538	450	
625	440	
700	430	
825	420	Boundary of National Trust area
950	410	
1100	400	
1350	390	Hollins Cross

Activity 39 *Plotting a profile*

Starting at the top of Mam Tor, the ridge path descends to Hollins Cross. Table 4 gives the distance from Mam Tor and the corresponding height of the path.

(a) Plot this information on a graph, using the grid in Figure 42. Plot *distance* on the horizontal axis and *height* on the vertical axis. You will need to decide on the scale of the axes, and label them appropriately.

(b) Give a detailed description of how the gradient of the path changes as you approach Hollins Cross.

(c) Work out the average gradient of the path for the stretch between Mam Tor and the edge of the National Trust area, about 0.8 kilometres away.

Figure 42 Blank grid

Using a graph, you can now represent the entire walk from Mam Farm to Losehill Farm. Figure 43 shows a graph of height plotted against horizontal distance for each of the locations listed on the route card in Section 3 plus two more to show up the feature of Back Tor more clearly. The graph is drawn as if the path of the walk has been 'unfolded' and laid out with the start on the left and the finish on the right. The data points are joined by straight lines to give an indication of the average gradient encountered at each stage of the walk. You can see the relatively steep climbs up to Mam Tor and the shorter section at Back Tor.

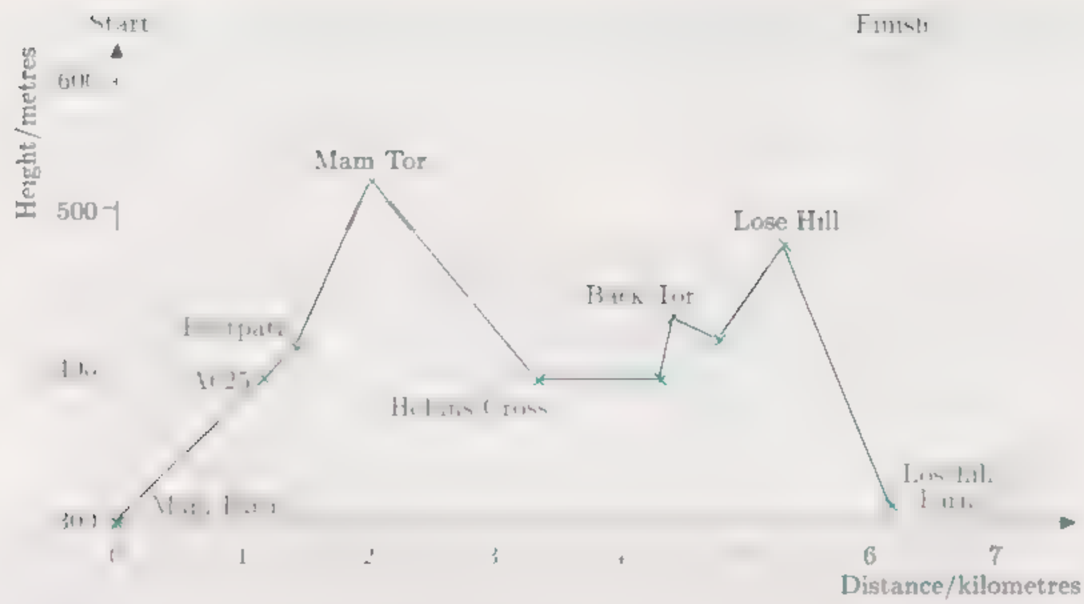


Figure 43 Plot of the profile of the walk

You may feel, however, that the plot is not quite right, that it gives the wrong visual impression. Even accounting for the straight lines, the profile of the walk appears to be far more jagged than the landscape you saw in the video. The slopes seem to be too steep. So what is wrong? What can you read out of the graph that will explain the impression?

The visual impression of a graph depends on the scales used on the axes. Notice in this case that a change of height of 200 metres is represented by the same length on the vertical axis as a distance of 2500 metres on the horizontal axis. Indeed, if the horizontal axis were drawn to the scale used on the vertical axis, it would be twelve and a half times longer.

Every picture tells a story, and graphical 'pictures' are no exception. One story of this unit concerns decisions about what to display, and how to display it.

The effect of the different scale is to exaggerate the vertical profile of the walk, so that the slopes appear to be far steeper than they are in reality. Changing the vertical scale so that it is the same as the horizontal scale would reduce the distortion, but the trade-off is that variations in height become small and difficult to read from the graph. When you plot a graph, be aware of the visual effect of using different scales on the axes.

Activity 40 *Seeing and believing*

Look back at the sketch graphs in this subsection. For each one, make a note about the effect of the scales of the axes on the visual impression given by the graph. What do you need to consider when choosing the scales of axes?

4.2 *Different slopes for different folks*

So far you have been sketching graphs and working out gradients using the information on your OS map. This is fine if you are planning a walk, but it is not the only way to come across gradient information. Road signs on steep hills usually give an indication of slope—but what do the signs mean and how are the figures they show calculated?

In the UK, the Department of Transport has used two different ways of showing the gradient of a road. Figure 44(a) shows the type of sign you might have seen a few years ago. The gradient is read as '1 in 10', and even if this did not mean much in itself, road users (especially cyclists!) would know by experience that this would be a steepish slope, more than 1 in 20, but not as steep as 1 in 5. Nowadays, gradient information is usually expressed differently, as a percentage, as in Figure 44(b).

Figure 44 Road signs showing (a) a gradient of 1 in 10 and (b) a gradient of 20%

▶ What do these figures mean?

A slope of 1 in 10 means that the road changes in height by 1 unit (which will be 1 metre if you are measuring distances in metres) for every 10 units you travel along the road. On the other hand, expressing a slope using percentages means that the change in height will be the stated percentage of the distance travelled. So a slope of 20 per cent, for example, means that the height of the road will rise or fall by 20 metres for every 100 metres travelled along it.

Activity 41 Road gradients

(a) What would be the change in height over a distance of 200 metres if the gradient is 15 per cent?

(b) What is the road gradient expressed as a percentage if the road changes in height by 50 metres over 300 metres?

Notice that there is a subtle difference between the way gradient was defined in the previous subsection and the way the road authorities define gradient.

Recall that the earlier definition of gradient was:

$$\text{gradient} = \frac{\text{change in height}}{\text{horizontal distance}}$$

The change in height is treated as positive if the height increases with distance, and negative if it decreases with distance.

This definition of gradient is shared by walkers and mathematicians, and you will come across it again in this course when you learn about graphs of mathematical relationships. In this section, this way of calculating gradients is called the mathematical gradient.

The road gradient is hence much easier to measure directly.

In contrast to the mathematical gradient is the road gradient, which is worked out by dividing the change in height of the road between two points by the distance measured *along the surface* of the road. So the definition is:

$$\text{road gradient} = \frac{\text{change in height}}{\text{road distance}}$$

But what difference is there between the mathematical gradient and the road gradient? You can find out by using some mathematics.

Once again use a model to help focus on the main features of the task. This should be familiar ground to you now; you used the OS map as a model of the country to predict distances and bearings, and Naismith's rule as a model of a walk to predict how long the particular walk would take without having to do it. Now here is a similar problem: to predict a road gradient from a map without having to go out and measure the road itself.

Recall the reader article in *Unit 1*: a cabbage is not a sphere, and a hill is not a triangle.

The model is very simple — in fact, it is just a triangle. You can think of the triangle in Figure 45 as the profile of a road going up a 'perfect' hill: there are no lumps or bumps and the road climbs with a constant gradient.

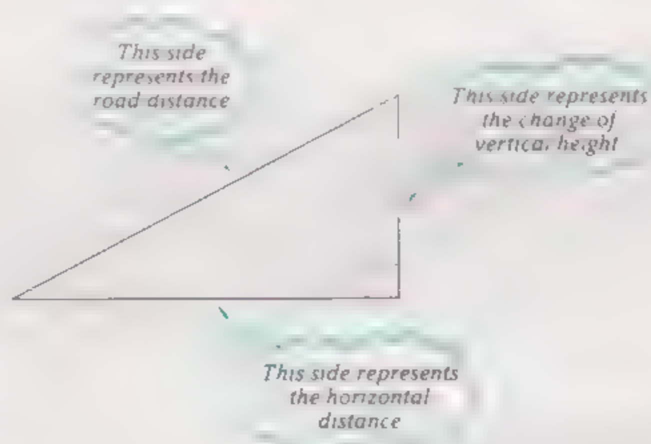


Figure 45 Triangle model of a hill

Notice that Figure 45 is a special sort of triangle. The angle between the horizontal side and the vertical side is 90 degrees, usually called a *right angle*. Since the angles in any triangle always add up to 180 degrees, the other two angles in Figure 45must add up to 90 degrees. In turn, this means that each must be less than 90 degrees. The right angle is therefore the largest angle in the triangle.

Now in any triangle, the bigger the angle, the longer the opposite side; the smaller the angle, the shorter the opposite side. You can see this is in the examples shown in Figure 46. Small angles such as A and B are opposite short sides, while larger angles such as C and D are opposite longer sides. In fact, in any one triangle the smallest angle is always opposite the shortest side, and the largest angle is always opposite the longest side.

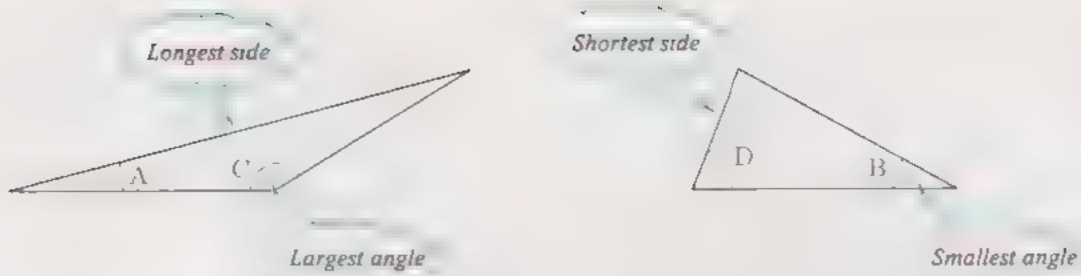


Figure 46 Examples of triangles

If a triangle contains a right angle, then this is the largest angle. So the longest side must be the side opposite the right angle. The longest side in a right-angled triangle is called the ***hypotenuse***.

Figure 47 shows a triangle model of a hill. The hypotenuse represents the road surface running up the hill. The horizontal side of the triangle represents the horizontal distance (the distance measured on a map) between two points on the road, and the vertical side represents the change in height of the road.

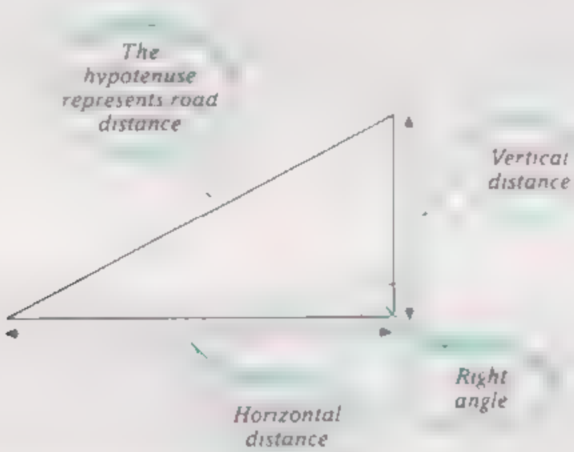


Figure 47 A right-angled triangle

The road gradient is the ratio of the change in height to the road distance for this hill. Information about the height change and the horizontal distance can come from an OS map, but not the distance measured along the road. Remember that because you are effectively looking straight down from above, the map will only tell you the distance along the road directly when the road is flat and the gradient is zero. Then the road distance and the map distance will correspond. But if the road is very steep, the horizontal distance shown will be small and the road will appear on the map shorter than it really is. In the extreme (and silly) case of a vertical road, the road would only appear on the map as a single point!

Working out the distance along the road corresponds in the mathematical model to finding the length of the hypotenuse in Figure 47. To carry out the calculation you can use a well-known result in mathematics, called Pythagoras' theorem.

Pythagoras' theorem

The result called *Pythagoras' theorem* is one of the oldest mathematical results known, and one of the most famous. The theorem has long been attributed to Pythagoras, a semi-mythical Greek of the sixth century BC who gave his name to a Mediterranean sect called the Pythagoreans. They followed a mystical philosophy inspired in part by pure mathematics, believing that numbers and number patterns were the key to understanding the world. However, it is clear from the clay tablets of nearly two thousand years BC that early Babylonian scribes knew about 'Pythagoras' theorem', and the result is also found in ancient Chinese manuscripts.

What Greek mathematicians may have contributed was the idea of 'proving' that the theorem always holds. In this form, it appears in the first book of Euclid's Elements, a work written in Egypt around 300 BC which drew together much earlier mathematical knowledge, and which has remained one of the world's mathematical best-sellers ever since.

Pythagoras' theorem relates the lengths of the three sides of a right-angled triangle to one another. Because of this unchanging relationship, if you know the lengths of any two sides, then you can always calculate the length of the third.

> The square of the hypotenuse is equal to the sum of the squares of the other two sides.

For Figure 47 this means that:

> The square of the length of the sloping side (the hypotenuse) is equal to the square of the length of the horizontal side plus the square of the length of the vertical side.

Using brackets and the symbol for the square of a number, this is more concisely expressed as:

$$\text{(length of sloping side)}^2 = \text{(length of horizontal side)}^2 + \text{(length of vertical side)}^2$$

Pythagoras' theorem crops up time and time again in mathematics. You will certainly come across it later in the course. The important point to remember is that it applies *only* to right-angled triangles, like our model of a hill.

Here is a numerical example. On your map, locate the kilometre square 1382. In that square you will find the line of the road from Castleton passing through Winnats. If you look carefully at the contours, you will see that, at its steepest section, the road rises by 50 metres over a horizontal distance of about 300 metres. This is a mathematical gradient of $50/300 \simeq 0.167$ (the road is quite steep). Now using Pythagoras' theorem you can work out the distance along the surface of the road.

$$\begin{aligned}(\text{distance along road})^2 &= (\text{horizontal distance})^2 + (\text{vertical distance})^2\\ &= (300 \text{ metres})^2 + (50 \text{ metres})^2\\ &= 92\,500 \text{ (metres)}^2\end{aligned}$$

The distance is found by taking the square root of both sides.

$$\text{distance along road} = \sqrt{92\,500 \text{ (metres)}^2} \simeq 304 \text{ metres}$$

So the distance along the surface of the road is 4 metres greater than the horizontal distance. The road gradient is calculated as:

$$\frac{\text{change of height}}{\text{road distance}} = \frac{50}{304} \simeq 0.164 \text{ (to three decimal places).}$$

The result for the road gradient is very close to the result of 0.167 produced by the calculation of the mathematical gradient. For all practical purposes—and for all reasonable slopes that you are likely to come across on the road—the two are effectively the same. Both measures indicate a slope of between 16 and 17 per cent, quite a steep climb if you are on a bicycle or on foot.

Work through the next activity to practise using Pythagoras' theorem.

Activity 42 *Using Pythagoras' theorem*

Suppose that from a map you have found that the horizontal distance between two points on a road going up a hill is 100 metres. Over this distance the height changes by 20 metres.

(a) Use Pythagoras' theorem to work out the distance measured along the road.

(b) Calculate the mathematical gradient and the road gradient of the road.

You should have found from the activity that the road gradient and the mathematical gradient were very close. But the difference between the results of the calculations becomes greater as the slope gets steeper.

▶ What would be the difference for a slope like that on the east side of Mam Tor?

At its steepest, the height of the ground on the side of Mam Tor rises by 50 metres over a horizontal distance of 25 metres, resulting in a mathematical gradient of $50/25 = 2$.

Calculated looking down from the top, it worked out to be -2. Here, it has been calculated looking up from the bottom.

Now to work out the corresponding 'road gradient'. The horizontal distance is 25 metres and the vertical distance is 50 metres, so the distance along the surface of the slope is given by Pythagoras' theorem.

$$\begin{aligned}(\text{distance along surface})^2 &= (\text{horizontal distance})^2 + (\text{vertical distance})^2\\ &= (25 \text{ metres})^2 + (50 \text{ metres})^2\\ &= 3125 \text{ (metres)}^2\end{aligned}$$

Taking the square root gives the distance $= \sqrt{3215\ (\text{metres})^2} \approx 56$ metres.

So the 'road gradient' here would be:

$$\frac{\text{change of height}}{\text{distance along surface}} = \frac{50}{56} \approx 0.89 \quad (0.9 \text{ to one decimal place}),$$

which is quite different from the value for the mathematical gradient of 2.

Perhaps you can see the problem with the road gradient definition. As the slope gets steeper the mathematical gradient gets bigger and bigger. The road gradient, however, gets closer and closer to 1. To take an extreme example, a mathematical gradient of 5 corresponds to a road gradient of 0.98. If the mathematical gradient increased from 5 to 10, then the road gradient would increase only from 0.98 to 0.995, a very small change to indicate a slope that is twice as steep in reality.

In mathematics, the slope or gradient of a graph is calculated in exactly the same way as a walker would predict the slope of a path or hillside from a map: by dividing vertical distance by horizontal distance.

4.3 SI units

In this unit, you have been dealing mostly with metric units: centimetres, metres and kilometres to measure distance, although occasionally you have come across mention of inches, feet, miles, and even cubits! Elsewhere, you may have come across different units of mass or weight, such as pounds and kilograms, or different measures of speed such as miles per hour, kilometres per hour and metres per second.

- ▶ Which units are you familiar with?
- ▶ Do you naturally think about lengths in terms of kilometres, metres or millimetres or are you happier with miles, yards and inches?
- ▶ When you are buying food do you buy in pounds and ounces, or do you work with kilograms and grams?

If you have grown up with both systems, you may be able to change from one to the other without too much difficulty.

This informal mixing of systems of units works fine for many people on a day-to-day basis, but it is not helpful for communicating effectively in larger mathematical, technological or scientific communities which run across national boundaries. In learning about mathematics you are learning to be a member—and to speak the language—of a wider international community.

Learning to be part of a community means learning its conventions and adopting particular styles of description and representation. In dealing with weights and measures, the widely accepted convention within mathematics, technology, science and commerce across the world is the *Système International d'Unités*, usually abbreviated as 'SI units'. The system of SI units is a metric system, and is the one you will be primarily using in this course.

In the SI system, a single basic unit of measurement is used for any given physical quantity. Metres are used to measure length, kilograms to measure mass and seconds to measure time. Each unit of measurement has its own symbol: for example, m for metres, kg for kilograms, s for seconds. From these basic units others are derived. The unit of area, for example, is the metre squared, often abbreviated to m^2.

Large and small quantities are handled by a system based on multiples of ten, which avoids the need for complicated conversions, such as 12 inches = 1 foot, 3 feet = 1 yard, and so on. A distance such as 90 000 metres, roughly the distance from the Open University campus to central London, is a large number to deal with. The SI system simplifies the handling of such a number by allowing a larger unit of distance, one thousand times the length of the metre: the kilometre, abbreviated to km. So the distance to London is referred to as 90 km.

The prefix kilo- has the same meaning whatever basic unit it is attached to: it means one thousand. So a thousand grams is called one kilogram (abbreviated to kg), while a kilohertz is one thousand hertz (kHz).

The *hertz* is a unit of frequency, equal to one cycle per second. The tuning scale on your radio is likely to be calibrated in kilohertz (kHz) or megahertz (MHz).

The same principle operates in the other direction: if a basic unit is broken down into one thousand equal parts, each of these is referred to as a *milli* unit. So a millimetre (mm) is a thousandth part of a metre, a milligram (mg) is a thousandth part of a gram and a millisecond (ms) is the thousandth part of a second.

A system of names and abbreviations has been devised to handle both very large and very small quantities. Table 5 shows some of the more common ones you may come across.

Table 5 Names and symbols used in the SI system (whatever the unit)

Prefix	Symbol	Number	Scientific notation
giga-	G	1 000 000 000	10^9 units
mega-	M	1 000 000	10^6 units
kilo-	k	1 000	10^3 units
milli-	m	0.001	10^{-3} unit
micro-	μ	0.000 001	10^{-6} unit
nano-	n	0.000 000 001	10^{-9} unit

When a prefix is put in front of a unit it produces in effect a new unit. So the unit of area produced by multiplying kilometres by kilometres is written as km^2 and can be read as $(kilometres)^2$ or kilometres squared or square kilometres. It should *not* be interpreted as kilo × $(metres)^2$, or a thousand square metres.

As you can see in Table 5, the prefixes generally change in steps of 1000, or 10^3. This is the preferred SI convention. A common exception, however, is the use of the prefix *centi-*, as in centimetre, meaning a hundredth part of a unit.

Activity 43 Measure for measure

(a) Express the speed 30 kilometres per hour in the basic SI units of metres per second.

(b) A human hair is about 0.05 mm across. Express this both in micrometres (μm), and in metres (m).

4.4 Estimating area

Ordnance Survey maps can be used to estimate areas as well as distances. Indeed, from an historical point of view, it was the effect of UK government legislation brought about by various Acts of Parliament in the 1920s and 30s on town planning, land registration, land drainage, slum clearance and land valuation that increased the demand for large-scale maps from which areas, among other things, could be accurately determined.

Area measurements made from maps are used by a wide range of national and local government agencies, water authorities, land management and environmental bodies. UK farmers too have turned to OS maps to make detailed estimates of the areas of land they propose to use for growing crops and the areas which will be left unused under the European Union's 'set aside' rules.

At the time of writing, (1994), farmers can be paid to 'set aside' a percentage of the land they would normally use for growing crops. The aim is to avoid large surpluses and to maintain market prices.

At a personal level, you too may want to work out areas. A scheme to preserve so many hectares of woodland, a plan to sell part of a local park for new housing, or a proposal to flood a valley to make a new reservoir all may prompt you to use some mathematics to find out exactly what area of land will be involved.

Blue grid lines divide your map into squares. Since the grid lines are spaced at 1-kilometre intervals the area represented by one square is 1 kilometre × 1 kilometre, or 1 square kilometre.

▶ What is the total area covered by your map?

You can count all the grid squares, or more simply note that there are nine grid squares along the bottom and eight up the side, giving a total of $8 \times 9 = 72$ squares. So your map represents an area of 72 square kilometres.

Activity 44 A feel for area

A square kilometre is quite a large area. Stop for a moment and think what would be included in a square kilometre centred on where you are now. You may find a sketch and a local map or street plan with a scale is helpful.

For smaller areas, a square kilometre (km^2) is too large, and a square metre (m^2) is a more convenient measure.

▶ How many square metres are there in a square kilometre?

1 km is the same as 1000 m, so the area represented by one grid square is

$$1000 \text{ m} \times 1000 \text{ m}.$$

Now 1000 metres is a quantity which reflects a measurement. Like all measured quantities, it is made up of a numerical part (1000) and a part giving the units of the measurement. When you calculate with such quantities, you need to make sure not only that the result is correct numerically, but that you write down the correct units as well.

What a measurement is, and how it can be represented and handled mathematically is part of an area of study called *metrology*.

Multiplying the numbers together gives the size (or magnitude) of the area

$$1000 \times 1000 = 1\,000\,000.$$

Since you started with metres, the corresponding unit of area is the *square metre* (or m^2). So the calculation is

$$1000 \text{ m} \times 1000 \text{ m} = 1\,000\,000 \text{ m}^2.$$

In other words, the area of one grid square is one million square metres. If you expressed this in scientific notation (try it on your calculator), it would be 10^6 square metres. You can now write down the relationship between 1 square metre and 1 square kilometre:

1 square kilometre = 1 000 000 square metres.

This can be abbreviated to:

$$1\text{km}^2 = 1\,000\,000 \text{ m}^2 = 10^6 \text{ m}^2.$$

Activity 45 Converting units of area

How many square centimetres are there in one square metre?

▶ How are areas shown on your OS map related to areas on the ground?

By subdividing the kilometre grid squares on the map to produce smaller squares, you can estimate the areas of irregular shaped features such as woods and lakes by counting the number of smaller squares and part squares inside the shape.

Each side of a kilometre grid square can be divided into ten equal parts using a ruler and pencil. Drawing in the horizontal and vertical lines corresponding to these subdivisions, as in Figure 48, gives $10 \times 10 = 100$ smaller squares.

▶ What (flat) area does one of these small squares represent?

Each kilometre grid square represents an area of 1 square kilometre, or $10^6 = 1\,000\,000$ square metres. So each of the 100 small squares will represent an area of $1/100 = 0.01$ square kilometre, or $10^6/10^2 = 1\,000\,000/100 = 10\,000 = 10^4$ square metres.

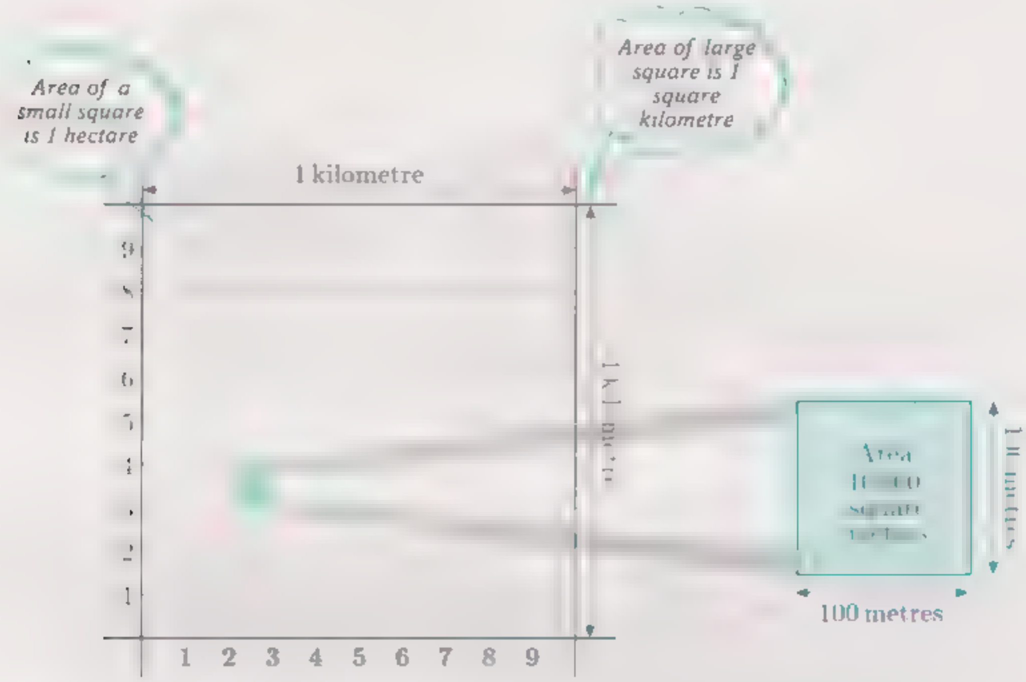


Figure 48 Subdividing a 1-kilometre grid square

An alternative way of looking at it is to note that each small square represents an area 100 m by 100 m, equal to 10 000 m². This area is called a ***hectare***, and there are 100 hectares in 1 km². The abbreviation for hectare is 'ha'.

Calculations of area made by subdividing grid squares and estimating the number of smaller squares covering a map feature will only be approximate. Not only is it difficult to estimate areas accurately at a scale of 1 : 25 000, but a feature, such as a woodland, may have changed since the map was produced. In urban areas, housing and industrial developments may quickly make a map out of date. If you need to know areas accurately, then you will need up-to-date large scale maps.

Activity 46 *Estimating area*

Towards the top of your map in the grid square 1489 is shown Gillott Hey Coppice. Estimate the area of this woodland in hectares.

According to a dictionary, a 'coppice' is 'a small wood of small trees grown for periodical cutting'.

The scale of a map relates measurements of distance on the map to distances on the ground. For your 1 : 25 000 map, this relationship can be written as a formula.

$$\text{distance on the ground} = 25\,000 \times \text{distance on the map}$$

How could you express the relationship between areas as a formula? The map legend does not tell us explicitly how areas are related. However, for level ground the area is proportional to the area on the map.

area on the ground = some number × area on the map

▶ What number should go into this formula?

The map scale shows that a distance of 1 *length* unit on the map represents 25 000 length units on the ground, in whatever units of length are used. Now use this relationship to work out the *area* scaling factor. 1 square unit on the map represents 25 000 × 25 000 = 625 000 000 square units on the ground. For example, a square 1 centimetre by 1 centimetre on the map represents a square 25 000 cm by 25 000 cm on the ground. In other words, 1 square cm on the map represents
$25\,000 \times 25\,000 = 625\,000\,000\ \text{cm}^2$ on the ground.

Map areas represent areas viewed directly from above. Like distances, the area of a sloping piece of ground will appear smaller the steeper the ground becomes. The area of a vertical cliff edge, to take an extreme example, would not appear on the map at all.

Thus if the map is drawn to a scale of

$1 : 25\,000$

then the areas will be related by a scale of

$1 \times 1 : (25\,000 \times 25\,000)$

or

$1 : 625\,000\,000.$

So the formula relating map and ground areas is:

area on the ground = 625 000 000 × area on the map

where both areas are measured in the same area units.

Activity 47 Predicting areas

(a) What area in hectares is represented by an area of 1 square centimetre on your 1 : 25 000 map?

(b) What area would 1 square centimetre represent if you were using a smaller scale 1 : 50 000 map?

Going from the map to the ground involves *multiplying* the map area by the area scaling factor. This involves a process of prediction: you are using the map as a model of the country and using mathematics to predict actual areas from the representation shown on the map.

You can also use the model the other way round and relate ground areas to map areas by *dividing* the ground area by the area scaling factor. This involves a process of representation: instead of dealing with the actual area you work instead with a representation of the area on the map.

Activity 48 Representing areas

The Ordnance Survey produces maps at a scale of 1 : 2500 for detailed planning. What area on the map would represent an area of 1 hectare on the ground? What would be the effect on the map area if the scale of the map were doubled to 1 : 1250?

In Subsection 4.1, you saw how height and distance information from a map can be used to plot a profile of the ground. The techniques used built on earlier skills for using and interpreting Ordnance Survey maps.

Subsection 4.2 looked at gradient as a mathematical description of slope. Two definitions of gradient are in use, one by map-users and one by road authorities. Mathematicians and map-users adopt the same definition of gradient: the ratio of vertical to horizontal distance. The mathematical gradient can be calculated directly from height and distance information on an OS map. The road gradient calculation needs information about the distance along the ground surface. Pythagoras' theorem was introduced to calculate the distance along the ground from the height and distance information given by the map.

Subsection 4.3 described the Système International d'Unités, usually abbreviated as SI units. This metric system uses metres to measure length, kilograms to measure mass, and seconds to measure time. It is the widely accepted convention used in mathematics, technology, science and commerce across the world.

Subsection 4.4 showed how grid squares can be used for predicting area. Each large square bordered by blue grid lines represents an area of 1 km^2. This area is a bird's-eye view, looking straight down from above, and does not take into account the distorting effects of hills and valleys. For smaller areas, it is convenient to break down the grid squares into 100 smaller squares. Each small square represents an area of 10 000 m^2, or 1 hectare. The conversion factor between map area and ground area is given by the square of the map scale.

Before starting on Section 5, take time to think about what you have learned in this section. If you have not been able to add notes to the Learning File activities as you have worked through the unit, take time to complete them now.

Outcomes

After studying this section, you should be able to:

- ◇ use the following terms accurately and be able to explain them to someone else: 'gradient', 'road gradient', 'positive gradient', 'negative gradient', 'Pythagoras' theorem', 'hypotenuse' (Activity 41)
- ◇ predict the gradient of a slope from a map (Activity 35)
- ◇ draw and interpret a profile from map data on height and position (Activities 36–40)
- ◇ use Pythagoras' theorem to work out the length of the hypotenuse of a right-angled triangle and hence predict road gradients from a map (Activity 42)
- ◇ relate an area measured on a map to an area measured on the ground (Activities 44, 45, 47, 48)
- ◇ express quantities such as length, time and speed using the SI system of units (Activity 43)

5 A calculated display

Aims The main aim of this section is to show how your calculator can be used for plotting line graphs, and for performing calculations using lists of coordinate data. ◇

A **list** of numbers may be the starting point for a plot of the profile of a hillside or for repeated calculations such as scaling map measurements, or working out times using Naismith's rule.

Plotting graphs point by point or repeating the same sort of calculation over and over again is tedious and time consuming when there are a lot of data. But help is at hand. Once you have one or more lists of numbers in your calculator, you can display line graphs and carry out repeated calculations quickly and with a lot less effort.

Using the calculator to display graphs means you do not have to find graph paper, pencil and ruler, draw and scale the axes, and plot the data point by point. But the calculator does not do all the work for you! You must still enter the data correctly, and choose the range over which you want the graph to be displayed. The calculator does not label the axes, so you must also remember what it is that the display represents.

Learning to display line graphs from lists of numbers is a first step towards being able to display graphs directly from mathematical formulas without first producing a table of coordinate pairs.

In planning the walk, you drew up a route card with grid references, distance and height information for each stage of the walk. Using Naismith's rule you were able to work out the time for each part of the walk separately, and so predict the time for the whole walk. This involved the repeated use of Naismith's rule. Using the facilities built into your calculator for doing arithmetic on lists of numbers, you can arrange for the repeated calculations to be done automatically.

Now work through Chapter 6 of the Calculator Book.

Outcomes

You should now be able to use your calculator to:

- ◇ display a line graph in an appropriate window, given a data list of coordinate pairs;
- ◇ perform calculations on the numbers in one or more data lists and display the results as a list.

6 Representations and mathematics

Aims This final section aims to bring together the different ideas and perspectives and asks you to present your own views on 'the map'. ◇

The unit began by introducing representation. As you have worked through this unit, you have probably realized the importance of representation to mathematical thinking—and indeed to thinking of any kind. But before being able to represent things, particular cases need to be considered and decisions have to be made—what to stress and what to ignore. This often means moving from taking a specific situation to one where you can begin to make generalizations. Mathematical thinking makes constant use of representations and the more aware you are of what is involved in designing and developing them, the easier it will become for you to engage in their use and interpretation.

So part of the process of representation is being able to generalize. You have been involved in this process many times as you have worked through the unit. Take contours as an example—although you considered particular cases of contour patterns from the OS map extract, you are now in a position to use that knowledge in a much wider sense—either interpreting other maps which use contours or other representations using different isograms. Other particular examples include measuring bearings, calculating areas, or drawing profiles. In every case, you were involved in solving a particular problem or considering a particular example. But, having completed that, you are now better able to use that specialized case in a more general sense, to see beyond the particular and use your knowledge in a range of different contexts.

In describing, interpreting and explaining things, metaphors or analogies are used extensively. Think about the different analogies quoted in this unit—using a bar magnet to help explain the Earth's magnetism or using Naismith's walking experience. Mathematicians call these 'models'. In every case, mathematicians single out for attention some narrow aspect or particular property of a more complex system. Models are constructed to help their thinking and solve a particular problem, and they represent the system freed from the additional complications that mathematicians have chosen to ignore; the model has been deliberately 'crafted' to do the particular job in hand.

Generalizing has to do with noticing patterns and properties common to several situations in a wide variety of contexts. Part of the process of generalizing often involves creating a 'shorthand' or special language. Mathematics frequently uses letters and diagrams; chemistry uses a system of letters, numbers and symbols to represent chemical compounds; music uses lines and symbols to represent notes and musical effects. These

languages represent both a particular case and a more general situation, and in every case certain features are stressed and others ignored.

With practice and experience you will become more adept at generalizing for your own learning. Generalizing involves noticing things that are common to numerous examples, and ignoring features which seem to be special to only some of them. The best way to learn about generalization is to try it, so take every opportunity to interpret and reformulate other people's generalizations as your own.

On the audiotape, there are a number of different perspectives about maps from a variety of people. You will hear the following.

The map and the geographer.
The map and the designer.
The map and Helen aged $8\frac{3}{4}$.
The map and the mathematician.

Now listen to band 3 of Audiotape 2

Activity 49 *The map and the MU120 student*

Look back at your notes for Activity 2—they may help you here.

After listening to the tape, provide your own perspective for a general audience on

The map and the MU120 student.

Before you start to write, take some time to plan what you want to say. Which key points do you want to bring out? For instance, you may wish to talk about the mathematics you can now see in a map, or how your reading of maps has changed from the beginning of the unit. Perhaps you want to look at stressing and ignoring or the different ways that features are represented on maps and how symbolic representation is also part of learning mathematics. Whatever you decide to talk about, try to focus on one or two aspects as the other speakers have done. On the tape, the descriptions are quite short—around 250 words, so make your commentary a comparable length. Prepare your plan and then complete your commentary on paper and keep it in your Learning File. If you have a recording facility, you might like actually to record your script.

Activity 50 *Reviewing your progress*

Now that you have completed this unit, try to allocate some time to thinking about your own progress so far. Use these questions to help you form your response.

(a) First, complete Activities 2 and 5 that you started in Section 1. Are you surprised by the range of mathematical techniques and activities you have covered as you worked through the unit? A number of key terms have been introduced in the unit. Take time to record the definitions for your handbook.

(b) You have used a number of course components to study this unit, including an OS map, a video band, reader articles, audio sequences and the calculator. You may also have attended a tutorial session or worked with other students.

Think about all the components you have used as part of your learning in this unit. For each one, try to say *how* it helped you to learn the mathematics and to come to grips with the ideas.

There is a printed response sheet for this activity.

(c) From your study of the course so far have you found some components to be more effective than others in helping you to learn? Which components are they, and why do you think they have been more effective?

(d) Look back at your planning sheet for Activity 1, and review your study targets.

Outcomes

You should now be developing skills at:

◇ setting and reviewing study targets in response to changing circumstances (Activity 50);

◇ using 'review activities' as part of learning;

◇ using different activities and media to focus your work, producing written, spoken and diagrammatic material as necessary (Activity 49);

◇ summarize, generalize and communicate your views on a mathematical topic such as maps (Activity 49).

Unit summary and outcomes

This unit has been about representations and relationships. To convey meaning, a representation must be read and interpreted, and to do this you need to be able to make sense of the symbols, conventions and styles that are used. Mathematics itself can be viewed as a language of special representations—numbers, symbols, diagrams—used to express particular ideas and relationships. The context of the unit focused on maps and the mathematics embedded in them.

Section 1 looked at some of the different ways maps are made to represent the world, and ways they can be read and interpreted. A map is a complex cluster of symbols, and contains within itself a rich structure of mathematical relationships as well as references to cultural assumptions and to cartographic tradition and practice.

Section 2 brought out some of the mathematical ideas embedded in both the construction of an Ordnance Survey map and its representation of the world. These include a two-dimensional coordinate system for specifying position, scales to convert between dimensions on the map and on the ground, and contour lines and special symbols to convey meaning about features of the built and natural landscape.

Section 3 focused on mathematical models—a theme which runs all through this unit. Mathematical models use mathematics to represent relationships in the real world, and are used in this unit to predict distances, compass bearings, slopes, the time that a walk will take. The predictive models bring together appropriate general mathematical relationships and specific items of information from the map.

Section 4 continued the modelling theme and discussed the representation of area on a map and the mathematical definition of gradient. It introduced Pythagoras' theorem, a well-known mathematical result, to relate the lengths of the sides of a right-angled triangle in a comparison of different definitions of gradient. You also saw how height and distance information can be presented graphically—and used graph-drawing techniques to provide a visual impression of the profile of the ground.

Section 5 provided exercises and examples for you to use with your calculator, bringing modelling and graphical skills together.

Finally, Section 6 looked again at the themes of representation and generalization that run through the unit, reminding you that mathematical thinking makes constant use of these concepts. The section provided an opportunity for you to present your own views both about what maps mean to you and how you feel your ideas have changed since the beginning of the unit.

The unit has looked at some of the mathematical ideas embedded in maps. It also pointed out to you that *any* representation of the world stresses

some features but ignores others. Keep this in mind when you meet other symbols, graphs and diagrams in the course. Learning mathematics successfully involves feeling more confident about handling symbolic, graphical and numerical conventions for representing ideas and relationships.

Outcomes

You should now be able to:

- ◇ discuss and describe the four *properties* (true shape, equal area, accurate distance, consistent orientation) which can be preserved in a transformation from a 3D to a 2D image;
- ◇ state the likely purpose of a map from the information it contains;
- ◇ identify features in a representation or mathematical model that are stressed and those that have been ignored;
- ◇ understand that a map is a symbolic representation of reality, and has a range of mathematical ideas embedded within it;
- ◇ use grid references to specify a location on an OS map;
- ◇ convert measurements of distance made at one scale to their corresponding values at another;
- ◇ explain how contour line patterns are used to represent three-dimensional shapes on a two-dimensional map, and give some examples of contour patterns;
- ◇ make notes from a videotape band, with the aim of giving an accurate description to others;
- ◇ use a protractor to measure grid bearings on an OS map;
- ◇ predict a compass bearing given a grid bearing and information about magnetic north, and vice versa;
- ◇ fix a location using the bearings from two other points;
- ◇ use a word formula to express a relationship, as in Naismith's rule;
- ◇ predict the mathematical gradient of a slope from a map;
- ◇ use Pythagoras' theorem to work out the length of the hypotenuse of a right-angled triangle and the road gradient;
- ◇ relate an area measured on a map to an area measured on the ground;
- ◇ express quantities such as length, time and speed using the SI system of units;
- ◇ draw and interpret a profile from map data on height and position;
- ◇ display on your calculator a line graph in an appropriate window given a list of coordinate pairs;

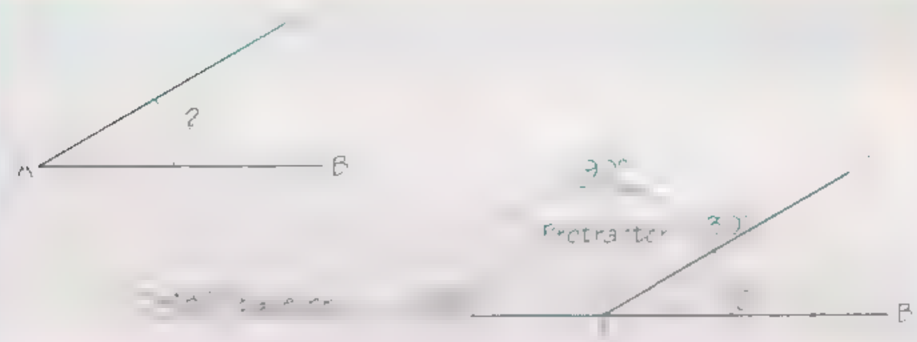
- perform calculations on the numbers in one or more lists entered in your calculator, and display the results as a list
- use the following terms accurately and be able to describe their meaning and use to others: 'grid system', 'map scale', 'legend', 'key', 'contour line', 'gradient', 'road gradient', 'positive gradient', 'negative gradient', 'perpendicular', 'Pythagoras' theorem', 'hypotenuse', 'grid bearing', 'compass bearing', 'grid north', 'magnetic north', 'mathematical model'.
- set and review study targets in response to changing circumstances
- use 'review activities' as part of learning.
- use different activities and media to focus your work, producing written, spoken and diagrammatic material as necessary.
- read materials for a purpose and extract the appropriate information.
- summarize, generalize and communicate your views on a mathematical topic such as maps

Appendix: Using a protractor to measure angles

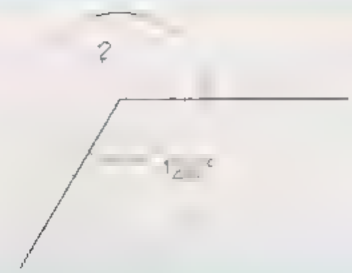
This appendix helps you to use a protractor.

Now listen to band 2 of Audiotape 2.

Frame 1



Frame 2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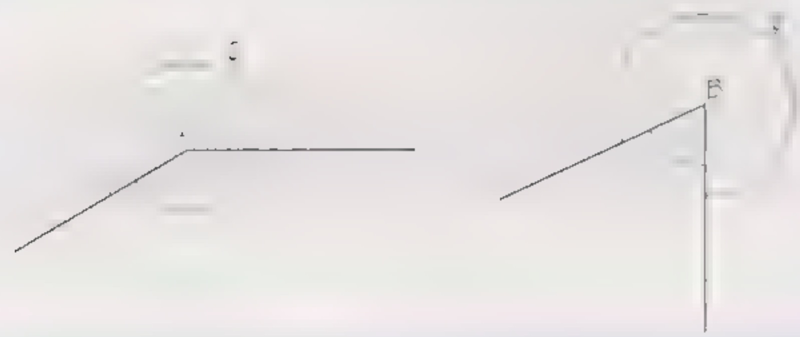
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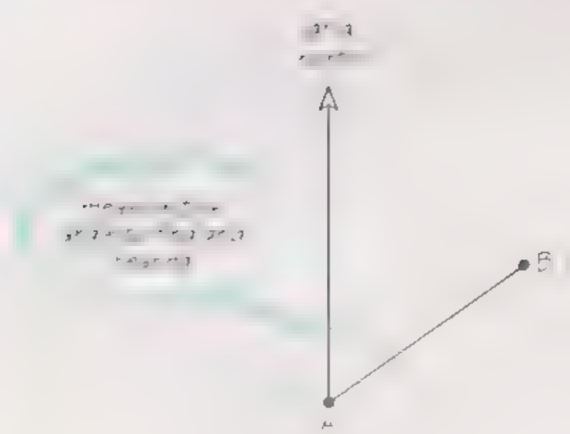
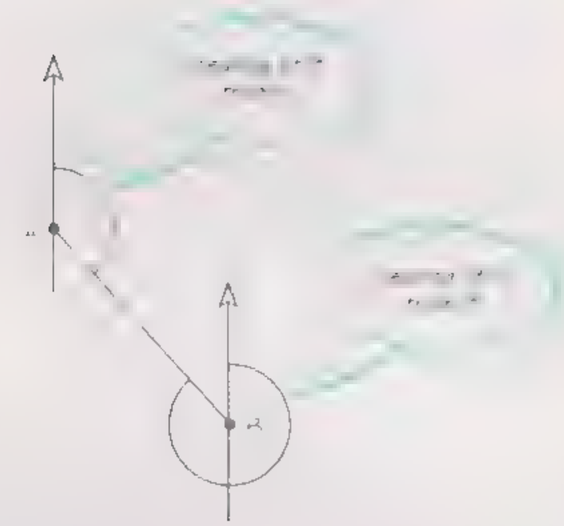
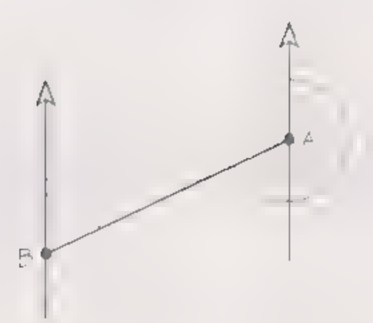

B

Frame 6



Frame 7



Comments on Activities

Activity 1

Now that you have some experience in studying on this course, it is a good time to review how you are planning and organizing your time.

After a while, you may feel it unnecessary to record a schedule formally, but it is often useful to jot down a rough plan which you can use to monitor and review progress.

Using assessment as part of your learning is important and using feedback constructively helps you to identify areas for improvement. Try to allocate enough time in your schedule to complete the assessment and work through any feedback comments.

To complete the last question of this activity, refer back to the skills audit you completed as part of *Unit 5*. Remember reviewing your progress is not time wasted. It can help you to assess where you are and what you want to achieve.

Activity 2

Everybody will probably have noticed varied and particular things about each map—there is no one single answer. For instance, someone who lives in Paris and studies with the Open University may be mildly interested to see the whereabouts of the central campus of the OU within Milton Keynes. Whereas someone else might be more interested in the way most, but not all, of the roads in Milton Keynes are on a grid system. But someone who is to visit the OU next week could use this map to help them get from the station to the campus.

You may initially notice that map (b) is an extract from a published Ordnance Survey map and a comment about such a map may include something about reliability or accuracy. If you enjoy walking, you might use this type of map to plan an interesting walk and then use it to help you to navigate on the walk. You may have noticed that the map contains information about the shape of the land—that is, flatness, steepness, the presence and position of river valleys, and so on. So you could use it to create a mental picture of the landscape and decide if it is an area you might like to visit. Or you may have noticed the names of places, historical connections, or the tourist attractions.

Did you notice the 'way up' (orientation) of maps (c) and (d) and were you surprised? The orientation of these maps may have told you something about the people who created them. You might use them to think about how the world looks from different points of view.

Map (e) is the sort of sketch map which many people draw to help somebody follow a route to a particular place, but like some of the other maps, the map itself can tell you something about the person or people who designed and drew it, and what they consider to be important landmarks.

And how do you know all these things? You have looked at each of the maps, the symbols, squiggles, and lines on each of them and made an interpretation about what they mean.

A printed sheet is provided for your notes as an ongoing record.

Activity 3

When designing and producing a map, there are always decisions to be made about which elements of the 3D-situation you wish to preserve in 2D. These elements can include the following.

- Compass direction: preserving direction means that straight-line directions on the map (the 2D-image) correspond to those on the actual ground.
- Distance: preserving distance means that the whole map is drawn to the same scale so that equal distances on the map correspond to equal distances on the ground.

- Area: preserving area means that the whole map is drawn so that equal areas on the map correspond to equal areas on the ground.
- Orientation: consistent orientation means that the whole map is drawn so that orientations on the map correspond to the same orientations on the ground.

  You may have thought of examples of transformations from 3D to 2D, such as television, cinema, ultrasound scan, photograph, X-ray image, portrait, contour map.

In the article, published in the *Times Educational Supplement* (a newspaper widely read by school teachers in Britain), the 'error' has come about by including a vertical north arrow but not a graticule on the printed map. In fact, the north arrow should align with the lines of longitude—by including them, the error could have been avoided and readers would have an improved representation of direction.

We have attempted not to fall into a similar 'ditch': a background grid or graticule (latitude and longitude) is included on the world maps.

Activity 4

(a) Your answer to this will probably depend upon exactly where you live, but you would probably try to draw angles correctly and at least part of the map to scale. However, some parts might not be to scale in order for you to get in all the important features of her journey from the bus stop. So scale may vary, and detail too. There might not be a need to have north at the top, it might for instance, be more convenient to have the road pointing towards the top. However, you would need to have some clear indication of scale and orientation, so that she started off in the right direction from the bus stop, and would be able to estimate journey time.

(b) Preserving angles (and directions) would be very important and, as far as possible, drawing the whole map to scale would be desirable. North cannot be at the top, because the North Pole would be in the middle of the map. So an alternative method of orientating the pilot would be needed. The lines of latitude and longitude should be useful here.

(c) All the above principles are important and north is conventionally at the top. However, distances and angles on the map cannot reflect the changes in height of the ground, the third dimension here. Contour lines are used to indicate heights and are discussed later in the unit.

(d) There may be a conflict here between wanting to preserve angles, distances and areas in mapping a sphere onto a flat sheet of paper. The choices that you make will depend upon your own cultural background and your own priorities. You could mark the relevant journeys on any of the world projections which you have seen earlier in this section, or use different ones.

Activity 5

As you work through the unit, continue to add notes to this activity. Take care to plan how you will present the information in an appropriate format and use a structure and style to emphasize meaning. This activity provides you with the opportunity to identify the mathematical aspects of the problems presented in the case studies, and to provide evidence of your ability to extract information for a particular purpose and summarize the information extracted.

Activity 6

This is a difficult and sensitive point to address, but may be partly explained by the medical profession's concern with confidentiality. Mapping may, for instance, target particular areas to such an extent that groups of people or individuals may be identified. However, world maps of the occurrences of different modes of transmission of the AIDS virus are now being used by researchers into the origins and spread of the disease.

Activity 7

The term 'tactile mapping' refers to any map in which touch is the predominant interpretative sense, but it is usually confined to those maps which are specially designed for blind and partially-sighted people.

A map that is designed to be read by the finger has a relatively poor resolution compared with one that is designed to be read by eye. Thus, a radical change of approach is needed on the part of the map designers. Rather than trying to include as much information as possible, the tactile map designer must exclude everything that does not contribute directly to transmitting the required message. As a result, the purpose of the map becomes more restricted and more significant in terms of its design.

The purpose of the map you have just designed was to help somebody to find their way about a strange place. This is one common purpose of maps. However, there are many others.

Activity 8

Wood's article reinforces some of the messages in this section. He comments that anyone who fails to see that a map is 'a weapon disguised as an impartial survey of the way things are' falls victim to it. He was reflecting on the fact that maps are used to redefine the world and not to represent it. Images of the world, however accurate, are bound to stress certain features and ignore others. And when you interpret maps, or indeed any image, you need to be aware of how representations were produced—for instance, what data were collected, who the map was created for, the purpose it was designed for, and so on.

Activity 9

If you have not used an Ordnance Survey map before, you may need to work through this section quite slowly, perhaps coming back a second time to some topics to make sure you understand them. However, if you are already familiar with OS maps, then the section will give you a chance to practise your skills, but now with an eye for the mathematical aspects of map reading.

Look quickly through the section, noting which topics will be new to you. Now look back to the schedule you completed in Activity 1. This is a good time to monitor your progress. Do you want to change anything?

When you have worked through the section, look again at your schedule and note down any disruption to your plan. How did you overcome it?

Activity 10

Using Figure 18, the grid squares are:

(a) *NN* (200 700)

(b) *TG* (600 300)

The grid references are:

(c) 300 1000 (*HY*)

(d) 500 200 (*TL*)

Activity 11

The grid reference is made up of two sets of numbers, each with the same number of digits. 127 is the easting and 836 is the northing.

The digits 12 of the easting and 83 of the northing locate the south-western corner (bottom left) of a 1-kilometre square. This point is where the vertical grid line with an easting of 12, read along the top or bottom of the map, crosses the horizontal grid line with a northing of 83, read from either side of the map. Locate the grid square which has the easting of 12 on the left, and the northing of 83 running along the bottom.

The given location lies inside this square. Now find the smaller 100-metre square defined by the digit 7 of the easting and the digit 6 for the northing.

By eye, or using a pencil and ruler, divide each of the sides of the 1-kilometre grid square into

10 divisions. (By drawing horizontal and vertical lines on the points you have marked, you could now, if you wished, subdivide the 1-kilometre square into 100 smaller squares, each representing 100 metres by 100 metres.) Count seven divisions from the left along the bottom of the square and draw a vertical line. Now count six divisions from the bottom up the left-hand edge and draw a horizontal line. The lines cross at the south-western corner of a 100-metre square. In that small square you should find the small triangular symbol used to indicate the triangulation pillar (used by the Ordnance Survey's surveyors) at the top of Mam Tor.

Activity 12

The grid references are as follows.

Hollins Cross	136845
Back Tor	145849
Lose Hill	153853

Activity 13

The map references correspond to the following.

(a) 172834 church

(b) 181832 railway station

(c) 123832 picnic site

(d) 140866 youth hostel

(e) 187851 spot height (from a ground survey) of 462 m at Winhill Pike

Activity 14

Children's playgrounds are not marked—although golf courses are. Bus routes are not marked—although the railway is. Remains of castles are marked, but day-care centres are not. There is no indication what the works to the south-east of Castleton produces or what it looks like or sounds like or smells like, all of which may be important to the people who live in the area, although perhaps less so for tourists for whom the map is ostensibly produced. The map shows what the Ordnance Survey considers to be relatively permanent features of the landscape—but there seems to be far less information directly relating to the *people* who live in the area.

Activity 15

(a) 12 centimetres on the map represents an actual distance of
$12 \times 25\,000 = 300\,000$ centimetres.
There are 100 centimetres in 1 metre, so the path is $300\,000/100 = 3000$ metres, or 3 kilometres long.

(b) A distance of 7.5 kilometres is 7 500 metres, or $7\,500 \times 100 = 750\,000$ centimetres. On the map this would be represented by a length of $750\,000/25\,000 = 30$ centimetres.

Notice that going from the map to the ground you *multiply* by the scale (which is bigger than 1), because you are going from the smaller map representation to the larger actual distance. Conversely, you *divide* by the scale when you go from the ground to the smaller map.

Activity 16

You should have found that the map distance was about 4.5 centimetres. This distance represents $4.5 \times 25\,000 = 112\,500$ centimetres on the ground; 112 500 centimetres is equal to $112\,500/100 = 1125$ metres, or 1.125 kilometres. Alternatively, you could use the fact that 4 centimetres on the map represents 1 kilometre on the ground. So a map distance of 4.5 centimetres represents a ground distance of $4.5/4 = 1.125$ kilometres. You might round this distance to 1.1 kilometres.

Activity 17

Hollins Cross to Back Tor is about 4.3 centimetres on the map, corresponding to about $4.3/4 = 1.08$ kilometres (to two decimal places) on the ground.

Back Tor to Lose Hill is about 3.5 centimetres on the map, corresponding to about $3.5/4 = 0.88$ kilometres (to two decimal places).

Lose Hill to Losehill Farm is about 3.4 centimetres on the map corresponding to about $3.4/4 = 0.85$ kilometres (to two decimal places).

The total distance from Mam Farm to Losehill Farm on the map is about 24.6 centimetres, corresponding to nearly 6.15 kilometres on the ground. If you add up the distances in kilometres in Table 1, you will get 6.17 kilometres. The slight difference is caused by rounding up or down in the conversion between centimetres and kilometres. Either way, the walk is about 6.2 kilometres long.

Activity 18

The isobars are drawn at intervals of 8 millibars (mbar). The line starts at the left-hand side of the map in a region of relatively high pressure: 1008 mbar. The pressure then falls rapidly to a minimum of 968 mbar in the North Atlantic, rises again to 984 mbar before falling again slightly to 976 mbar. The pressure then rises to 1032 mbar to the west of Europe before slowly falling again to a minimum of 984 mbar in the south of Finland.

Note *how* variation is described. It is important to be as clear and precise as you can. In this case, pressure and location details are provided whenever a change is identified.

Activity 19

The figure shows a sketch of the path from Back Tor to Lose Hill. Midway between Back Tor and Lose Hill the contours are relatively widely spaced. For a short distance around the 417-metre spot height the path is level. Approaching Lose Hill the line of the path cuts the contour lines indicating an increase in height. The contours also get closer together, indicating that the steepness of the path increases. For a sketch profile, as here, you do not need to be too concerned with including every detail relating to height. A sketch is, as the word implies, to give a visual impression.

Activity 20

There are no comments for this activity.

Activity 21

Every student will have something slightly different but here are some ideas.

You could use the compass extracts from the video together with a protractor, a compass and a map. You could use the video extracts of Man Tor and Hollins Cross with the computer animation and description of contour lines. However, a model of some sort with 'contour' strings stuck on might make the points too (plasticine or raw pastry dough makes a good cheap model). You could liken a saddle point to a horse's saddle and have pictures of both.

The main points should include the following.

- Horizontal angles and directions are preserved on the map, but magnetic north (which changes continually) needs to be taken into account for accurate bearings.
- Align the magnetic needle of the compass with the magnetic north on the map.
- Heights are represented by contour lines on the map; when the lines are close together the slope is steep.
- It is important to notice directions in which contour heights are increasing and decreasing to identify features like peaks or dips and which way the 'saddle' goes.

Activity 22

The bearings are: 45 degrees for north-east, 135 degrees for south-east, 225 degrees for south-west, and 315 degrees for north-west.

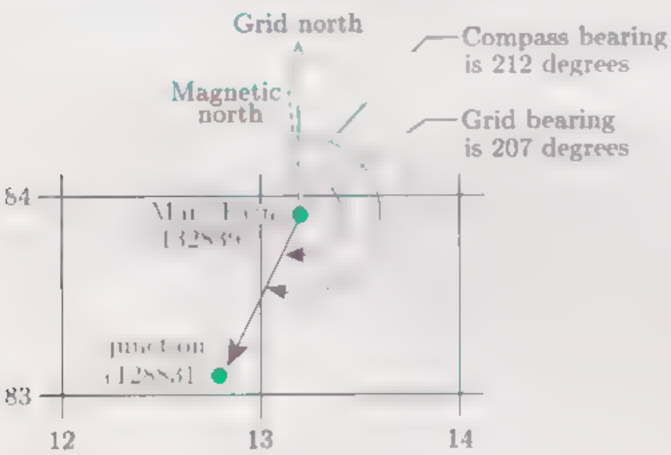
Activity 23

In 1994, the angle between magnetic north and grid north was estimated to be decreasing by about 0.5 degrees every three years. At this rate, magnetic north will have moved east by 5 degrees after ten times this period, that is 30 years, coinciding with grid north in about 2024.

Activity 24

30 degrees. See the comments in the text following the activity.

Activity 25



The figure shows the bearings. From the map, the junction lies on a grid bearing of about 207 degrees. Going from map to ground means adding 5 degrees to the grid bearing,so the compass bearing should be 212 degrees.

From the contour lines, the junction is at a height of about 390 metres. But the line drawn from near Mam Farm (which is at 310 metres) just grazes the 400-metre contour. In other words, the ground rises to 400 metres between Mam Farm and the junction, blocking the line of sight. So the junction cannot be seen from where the walkers were standing.

Activity 26

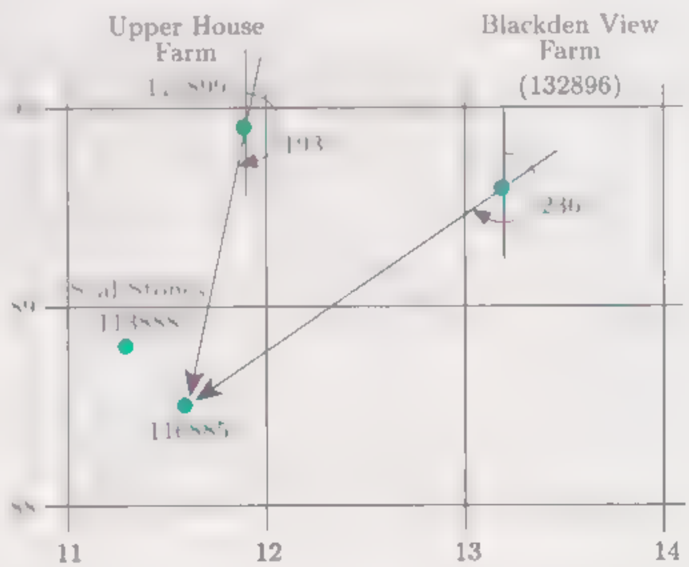
Measuring the angles from grid north using a protractor gives the bearing of Upper House Farm as 28 degrees, and of Blackden View Farm as 67 degrees.

Since you are going from the map to the ground add 5 degrees to each. This predicts that the compass bearings, *when measured in the physical world*, will be 33 degrees and 72 degrees respectively.

Activity 27

The reverse compass bearings are $18 + 180 = 198$ degrees from Upper House Farm, and $61 + 180 = 241$ degrees from Blackden View Farm. These are converted to grid bearings by removing 5 degrees from each, giving a grid bearing of 193 degrees from Upper House Farm and a grid bearing of 236 degrees from Blackden View Farm.

If you draw the lines corresponding to these bearings on your map, as shown in the figure below, you should find that they cross at about 116885, indicating that you are a little further east than you had planned.



Activity 28

number $+180 - 360 =$ number $- 180$

Activity 29

The grid bearing is about 145 degrees. Adding 5 degrees to account for magnetic north predicts a compass bearing of some 150 degrees.

Activity 30

The word formula is developed in the text following the activity.

Activity 31

Over the 2 kilometre stretch, the ground rises by $517 - 300 = 217$ metres. Naismith's rule predicts that the time to walk this section will be:

$$\text{time in hours} = \frac{2}{5} + \frac{217}{600} = 0.4 + 0.36$$
$$= 0.76 \text{ hours, or about 46 minutes.}$$

Of course, the estimated time is only a guide, but it provides a general figure for planning purposes. Completing the walk in less than 46 minutes is doing well.

Activity 32

In your list you may have identified that Naismith's rule:

- stresses the same constant speed for walking across level ground and going down slopes;
- stresses a different constant speed when walking uphill.

But it also ignores:

- the fact that people get tired; it assumes that walking and climbing speeds will be the same throughout the walk;
- variations in what people will carry; it assumes that the weight of a rucksack has no effect on the walking speed of the person carrying it;
- weather conditions;
- ground conditions;
- differences in fitness and stamina between groups of walkers.

Activity 33

There are no comments for this activity.

Activity 34

From Back Tor to Lose Hill, the total climb is about 70 metres, over a distance of about 900 metres. Naismith's rule predicts that this stage will take $(0.9/5) + (70/600) = 0.3$ hours, or eighteen minutes.

The path down from Lose Hill to Losehill Farm is about 0.9 kilometres long. Naismith's rule ignores the height descended and predicts the time for this final stretch to be $0.9/5 = 0.18$ hours, or just over ten minutes.

Adding the times for these last two stages to the times already listed on the route card gives a time of 110 minutes, or 1 hour 50 minutes for the complete walk. Experienced walkers aim to estimate times to an accuracy of about 10 percent. This is about ten minutes in this case. So you should expect the walkers to reach Losehill Farm within about ten minutes of two hours.

From the start of the walk at Mam Farm to the finish at Losehill Farm the distance is nearly 6.2 kilometres and the total ascent is about 330 metres. Naismith's rule predicts that the time to complete the walk will be $(6.2/5) + (330/600) = 1.79$ hours, or 1 hour 47 minutes. Since the times quoted for each stage were rounded up or down to the nearest minute, this result supports the previous estimate of 1 hour 50 minutes.

Activity 35

(a) Measuring in metres the gradient is $100/300 = 0.33$ (to two decimal places); the ground rises 0.33 metres for every metre moved horizontally.

(b) The change of height in feet is $100 \times 3.281 = 328.1$ ft and the horizontal distance is $300 \times 3.281 = 984.3$ ft. So once again the gradient is $328.1/984.3 = 0.33$. If you measure in feet you will find the ground rises 0.33 feet for every foot moved horizontally.

(c) Measured in cubits, the change of height is $100/0.49 = 204.1$ cubits, and the horizontal distance is $300/0.49 = 612.3$ cubits. So the gradient is $204.1/612.3 = 0.33$. No surprises here. If you are one of the few users of cubits left in the world, you will find that the ground rises 0.33 cubits for every cubit moved horizontally.

Activity 36

There is no comment for this activity. The completed graph is in Figure 40.

Activity 37

Zero on the horizontal scale represents the position of Back Tor. The height of the path increases and reaches a peak of 430 metres just beyond Back Tor, before dipping to 410 metres about 300 metres further on. The graph then slopes up to the right indicating a positive gradient; the path continues up a slope to the top of Lose Hill. The graph is not a straight line, however, so the gradient is not constant. The line indicates that the path is steepest about 700 metres from Back Tor.

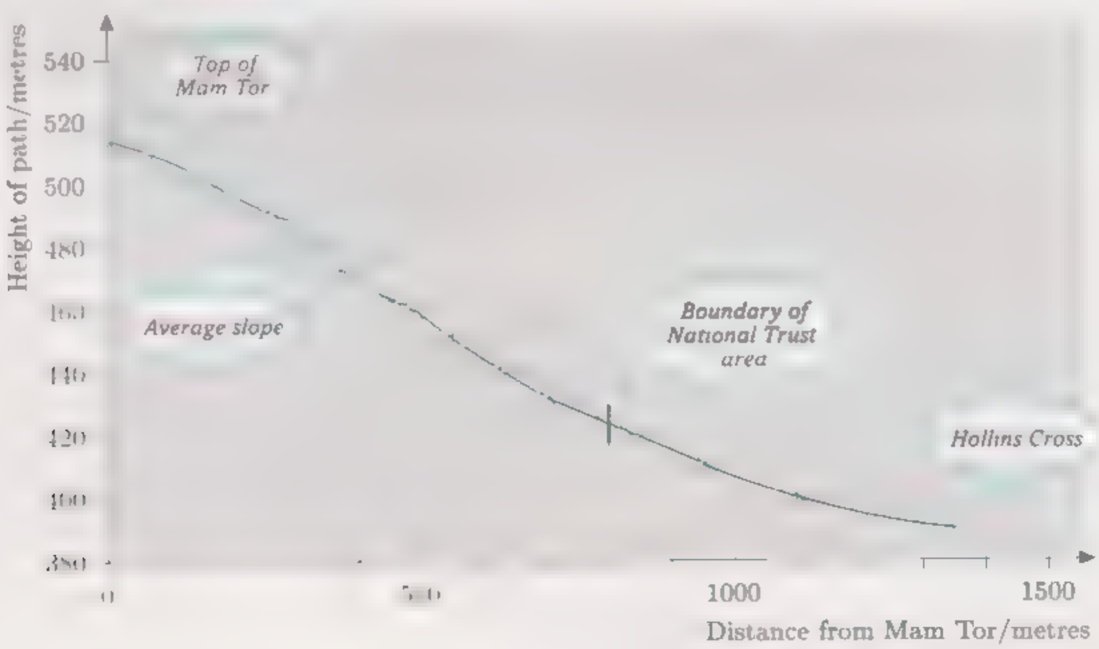
Activity 38

You can see from Figure 41 that the ground to the east of Mam Tor varies considerably from the average slope, shown with a dashed line. Between about 75 and 100 metres from the top the ground drops rapidly by about 50 metres. The gradient here is $-50/25 = -2$. That is, the height falls by 2 metres for every metre moved horizontally. This is extremely steep indeed. To get an idea of the slope you would face if you were climbing the Tor at this point, imagine you are standing about 1 metre away from a door in an average room. At this slope, the floor would rise to the top of the door over the distance between you and the door.

At other points, however, the story is quite different. For example, between about 250 and 350 metres due east of the summit the height changes by almost nothing. So here the gradient is $0/100 = 0$. A gradient of zero, therefore, means that the ground is level.

Activity 39

(a) The figure shows the plot of the distance and height data.



Profile of the path from Mam Tor to Hollins Cross

(b) The path slopes down from Mam Tor with an initial gradient of about -0.11. The height of the path drops fairly steadily for about 340 metres, then steepens slightly for a short distance before resuming the original slope. The slope becomes slightly less steep as the path crosses the boundary of the National Trust area, and continues to flatten out as it approaches Hollins Cross.

(c) Between Mam Tor and the National Trust boundary the path drops from 517 metres to about 425 metres, over a distance of some 800 metres. The average gradient of this stretch will be negative (because the height decreases as the distance from Mam Tor increases) and is about $-92/800 \simeq -0.12$.

Activity 40

Figures 39 and 40 exaggerate the steepness; Figure 41 does not, because the vertical and horizontal scales are the same.

Activity 41

(a) The change will be 15 per cent of 200 metres which is $\left(\frac{15}{100} \times 200\right) = 30$ metres.

(b) Expressing 50 as a percentage of 300 gives $\left(\frac{50}{300} \times 100\right)\% \simeq 17\%$.

Activity 42

(a) By Pythagoras' theorem

$$
\begin{aligned}
&(\text{distance along road})^2 \\
&= (\text{horizontal distance})^2 + \\
&\quad (\text{vertical distance})^2 \\
&= (100 \text{ metres})^2 + (20 \text{ metres})^2 \\
&= 10\,000 \ (\text{metres})^2 + 400 \ (\text{metres})^2 \\
&= 10\,400 \ (\text{metres})^2
\end{aligned}
$$

The distance is found by taking the square root:

$$
\begin{aligned}
\text{distance along road} &= \sqrt{10\,400 (\text{metres})^2} \\
&\simeq 102 \text{ metres.}
\end{aligned}
$$

(b) The mathematical gradient is:

$$\frac{\text{vertical height}}{\text{horizontal distance}} = \frac{20}{100} = 0.2$$

The road gradient is:

$$\frac{\text{vertical height}}{\text{road distance}} = \frac{20}{102} \simeq 0.196$$

The steepness of the road is nearly 20 per cent, or 1 in 5.

Activity 43

(a) 30 kilometres is 30 000 metres. One hour contains $60 \times 60 = 3600$ seconds. So the speed is $30\,000/3600 = 8.33$ metres per second (to two decimal places), sometimes written as 8.33 ms^{-1}.

(b) 1 mm is equal to $1000\,\mu$m, or 0.001 m. So 0.05 mm is equal to $0.05 \times 1000 = 50\,\mu$m. Working in metres, 0.05 mm is equal to $0.05 \times 0.001 = 0.00005$ m, or 5×10^{-5} m.

Activity 44

There are no comments on this activity.

Activity 45

Imagine a square 1 metre by 1 metre. A metre contains 100 cm, so in one square metre there are:

$100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2 = 10^4 \text{ cm}^2$.

In other words, there are ten thousand square centimetres in one square metre.

Activity 46

On the map, between five and six small squares (each representing 1 ha $= 10\,000 \text{ m}^2$) can be fitted into the coppice area. The actual area is therefore about 5 or 6 hectares.

Activity 47

(a) $1\,\text{cm}^2$ represents an area on the ground of $(25\,000)^2 = 625\,000\,000\,\text{cm}^2$. There are $100 \times 100 = 10\,000\,\text{cm}^2$ in $1\,\text{m}^2$, so the ground area is $625\,000\,000/10\,000 = 62\,500\,\text{m}^2$.

1 hectare is an area equal to $10\,000\,\text{m}^2$. So the ground area in hectares is $62\,500/10\,000 = 6.25\,\text{ha}$.

(b) At the smaller map scale, $1\,\text{cm}^2$ would represent an area of $(50\,000)^2 = 2\,500\,000\,000\,\text{cm}^2$ on the ground. This is four times the size of the previous area, so the ground area is $4 \times 6.25 = 25\,\text{ha}$, or $0.25\,\text{km}^2$.

Notice that *reducing* the map scale ratio by a factor of 2 from 1 : 25 000 to 1 : 50 000 leads to a four-fold increase (because $2^2 = 4$) in ground area for the same map area.

Activity 48

For a map at a scale of 1 : 2500, the area scaling factor is $1 : (2500)^2$, or 1 : 6 250 000. 1 hectare is $10\,000\,\text{m}^2$. So the corresponding area on the map is $10\,000/6\,250\,000 = 0.0016\,\text{m}^2$, or $16\,\text{cm}^2$. If the hectare is a square 100 metres by 100 metres, the area on the map is a square 0.04 metres by 0.04 metres, or 4 centimetres by 4 centimetres.

If the map scale is doubled, the map area will be decreased by four times. At a scale of 1 : 1250, 100 m is represented by $100/1250 = 0.08$, or 8 cm on the map. So 1 ha (100 m × 100 m) is represented by $8 \times 8 = 64\,\text{cm}^2$ on the map, an increase of 4 times.

So on the larger scale map, the same ground area is represented by a map area four times as large as the equivalent region on the map with half the scale.

Activity 49

You should have reviewed your ideas about maps. What do you want to say? What message did you want to get across? How have the speakers on the tape achieved this?

In planning your commentary, you may have noted down some ideas and then structured them in a logical order, concentrating on one or two main points—try not to overload or confuse what you want to say. Don't be afraid of deleting some of your planned points!

Remember that this will be heard by a general audience—people who may not be specialists in mathematics, and they are not reading it, but listening to it. Try to make sure your contribution is relevant and expressed in a suitable way for your listeners.

When you have completed your commentary you may wish to try it out on someone else—for example, another student, friend or partner.

Summarizing your own views about something is often very helpful in clarifying ideas and helping you to move forward in your own thinking. Taking time to review and reflect on what you have learned is not time wasted. It is time well spent in developing your understanding.

Activity 50

There are no comments on this activity.

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ISBN 0 7492 2232 8